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1826

1956

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FRED L. WHIPPLE

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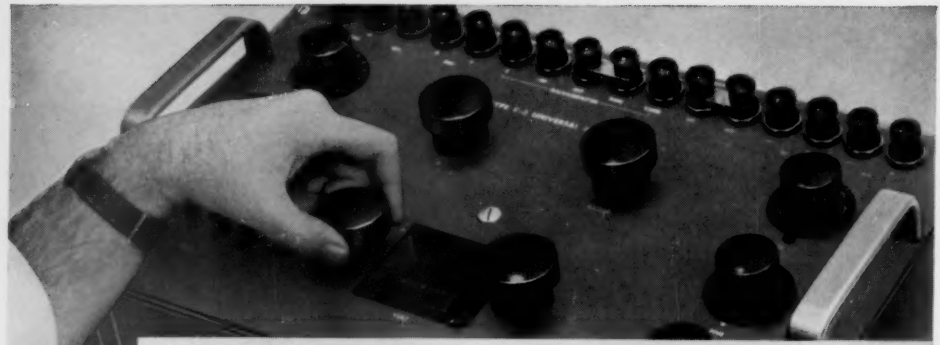

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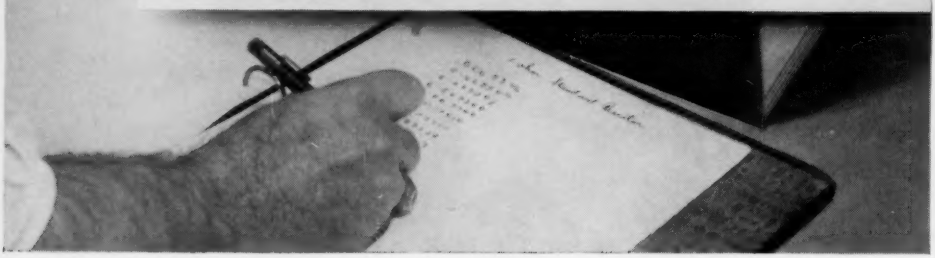

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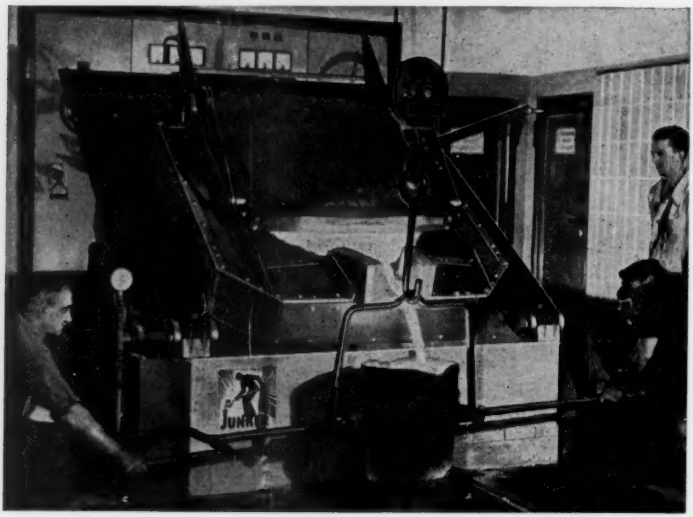

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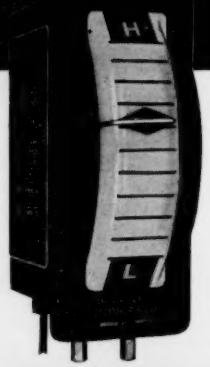

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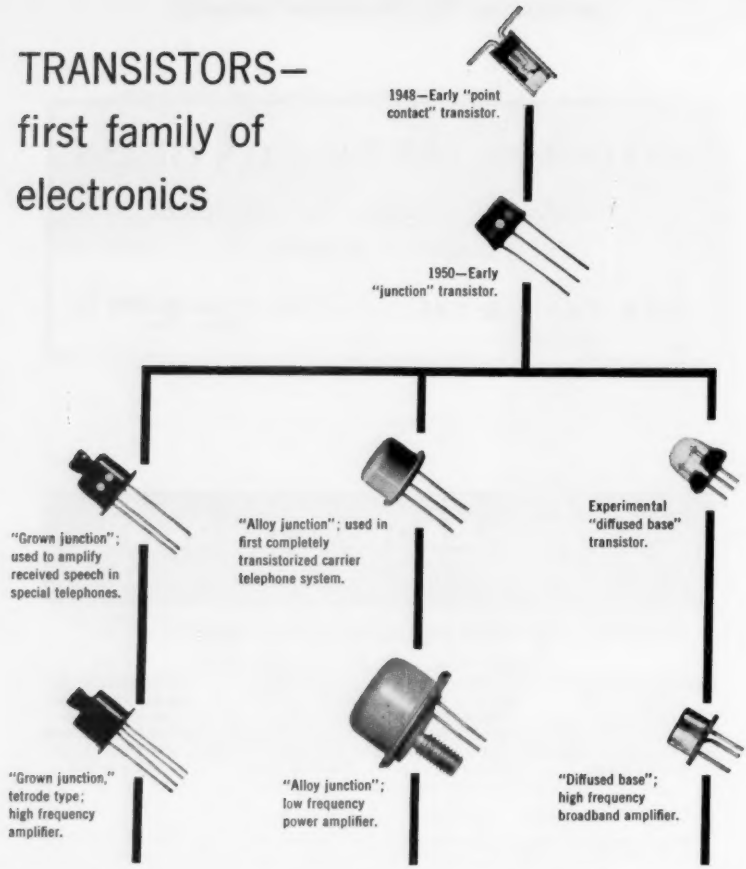
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Vol. 262 AUGUST, 1956 No. 2

THE SCIENTIFIC VALUE OF ARTIFICIAL SATELLITES *

BY

FRED L. WHIPPLE [1]

The subject of tonight's lecture represents to me an unparalleled situation in which there are more activity and more interest in anticipation of a scientific event than in the past history of science. I hope this is a good omen. Sometimes these events don't turn out so well, but in this case I think we are on good sound ground and we are truly going to see an artificial satellite moving about the earth before too long. I would not say exactly when, but within a year or two.

This is rather a remarkable time in the history of the world. One is impressed by the mushrooming of science, the accumulation of knowledge and its use for practical and sometimes impractical things. Today, coming down on the plane, I though about the various rates at which progress seems to take place. It must now be something like a billion years or so from the time we started out with simple molecules which developed into heavy proteins and small celled animals; then perhaps four hundred million years or so ago, sizable animal life appeared on the planet; and finally, a hundred million years carries us from the time of the early mammals to the place we are today. If you draw this curve, which seems to me very much like the curve of scientific progress during the last century or two, it is what we call an exponential curve. It keeps on getting steeper and steeper as it goes on. The scientific progress on this sort of scale will, I think, in the course of another 50 years or so, probably be as high above the surface of the earth as the satellite we want to talk about.

* Lecture delivered at the Stated Meeting of The Franklin Institute, Wednesday, April 18, 1956. The lecture summarized, for Institute members, the proceedings of the morning and afternoon sessions of the symposium on "Earth Satellites as Research Vehicles," sponsored by the Institute and attended by approximately 300 scientists and engineers earlier in the day. The Institute, under the sponsorship of the JOURNAL, published the full proceedings of the symposium, in June, as Monograph No. 2.

ED. Note: Dr. Whipple's paper was transcribed from a tape recording of his lecture.

[1] Director, Smithsonian Astrophysical Observatory, Cambridge, Mass.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.

Now where do we stand in regard to producing artificial moons which revolve about the earth—the artificial satellites? The subject of today's discussion has been what we might do with them scientifically; first, however, I'll take just a little time to discuss their present status and a bit of their history.

I may say that to the best of my knowledge, "flying saucers" do not represent a manifestation in this direction. All the evidence that I can find points against their being inexplicable. In other words, we have as yet no artificial satellites or space travel, although there are some who would like to interpret "flying saucers" as such.

The present satellite program, as you know, was announced by the President of the United States last July as part of our International Geophysical Year Program. The IGY, of course, is not a year or a measure of time, but a united program among the geophysicists of the world to make observations of the upper atmosphere, solar activity, the oceans and the other phenomena that are important in the study of the earth. The program was initiated by various international societies and in this country is sponsored by the National Academy of Sciences. The finances for it are provided by Congress through the National Science Foundation. In the IGY Panel on Rocketry, of which I have the honor to be Chairman, we developed a sub-panel with the code name, LPR. LPR, to us, stood for Long Playing Rockets—rockets that would go around the earth in a satellite orbit and therefore stay up longer than others. The LPR committee looked into the satellite potentialities, practical and scientific. We sent a favorable report to the IGY U. S. National Committee and at that point it rose out of my sight to the higher spheres in Washington, out of telescope range. You know where it eventually landed; the President announced our Satellite Program last July.

That program, as it has been set up, is controlled scientifically by the IGY committee of the National Academy of Sciences, and financially by the National Science Foundation. Our first step was to collect evidence to the effect that it was perfectly possible by means of modern rocketry techniques to put a small object into an orbit about the earth so that it would become an artificial moon. This is possible in terms of technology today, without new scientific break-throughs and new types of knowledge, but merely through development of what we now know.

It became immediately evident that only the military is now in a position to put such an object into an orbit and so the President assigned to the Department of Defense the task of doing two things: (1) putting a satellite into an orbit, and, very important, (2) proving that they had done so. That assignment in the Department of Defense was then given to the Naval Research Laboratory as Project Vanguard, which is headed by Dr. John Hagen, an Astronomer at the

Naval Research Laboratory. The Glenn L. Martin Company is the primary contractor for the rocketry, but the scientific aspects of this program are in the hands of a group called the "Technical Panel for the Earth Satellite Program" headed by Dr. Richard Porter of the General Electric Company, who spoke here this afternoon. I have the honor to be a member of that committee along with six or seven others of whom three were speakers today. That panel has the responsibility of deciding what scientific research can and should be done by means of the first artificial earth satellites.

I'd like to spend a little time describing this program and what is expected, because many of you may have read the newspaper accounts. It's a little difficult to determine from them what the exact situation is and how much to read between the lines. The intention of Project Vanguard is to put up into a satellite orbit an object weighing 21½ pounds, probably spherical (at the request of the scientific panel), with a diameter of about 20 inches. It should also be a good reflector, for a mirror-like surface is desirable on the satellite so that it can be observed optically. As to launching, this will probably be done with three-stage rockets. The first stage, which drops off, will be very large, comprising most of the mass and weighing something like 20,000 lb. It will go up to an altitude of about 40 miles, with a velocity of about 4000 miles per hour, and then fall off. The second stage will continue on upward to about 130 miles; the last stage will carry the small 21-lb. satellite into an orbit with a velocity of about 18,000 miles per hour. It will revolve about the earth in some 100 minutes. The plan is to put it into an orbit so that its minimum distance from the earth is 200 miles and the maximum distance is 800 miles.

Launching will take place from the Patrick Air Force Base at Point Canaveral, Florida. A little geometry will show how this fact affects the inclination of the satellite's orbit to the earth's equator and what regions of the earth it will pass over. Newton showed that a body moving around a spherical earth would move in a plane through its center. Consequently the satellite must start out in an orbit whose plane passes through the center of the earth and also through Patrick Air Force Base. If you can visualize these two planes—one the equator of the earth and the other plane going through a point at latitude 28° N—you will immediately see that they must be inclined *at least* by the angle of 28°, meaning that the satellite must perforce travel in an orbit that is inclined 28° or more to the earth's equator. Thus, as it goes around the earth it will pass over all latitudes between 28° N and 28° S. An attempt will be made to place it in a somewhat more inclined orbit, say about 40°, because then it would pass very nearly over Philadelphia! My guess, however, is 35°, which will still make the object somewhat southerly from Philadelphia when it's in the northern-most part of its orbit. The satellite starts out in a direction

parallel to the northern edge of the Bahamas, which stretch out toward the SE from a point in Florida.

There are some problems that limit the directions. People say, "Why not fire the Satellite in an orbit that goes up over the poles?" This would require a northerly (or southerly) original motion toward the poles from the launching site. However, the first and second stages of the rockets are going to go hundreds of miles and land in some spot not completely controlled from the launching site. It might be in the Carolinas or somewhere in Virginia, if we fired north from Patrick Air Force Base. But I'm afraid the people there would not appreciate having such objects falling down at random on their land. And the people in the Bahamas wouldn't like it, either. It's a little easier to get people out of the way when they are in ships on the water than it is when they are ensconced on land masses, so it's much better to fire out over the water. Therefore, a polar orbit from Patrick Air Force Base is highly unlikely. You can't fire south, either, without getting into trouble; so it doesn't look as though there would be a very high inclination for these first satellites.

A satellite itself moves around the earth in about an hour and forty minutes, during which time the earth turns on its axis about 25°. This means, then, that the earth is continuously spinning underneath this orbital path which a satellite follows, and so with every successive revolution of the satellite it passes over a region of the earth westerly from the one before, by a longitude difference of something like an hour and a half. As you know, it's about three hours in longitude across the country, three hours of time change, so two revolutions are more than enough to carry the orbital path westerly across the United States, as seen from the ground. The motion of a satellite is extremely rapid. If it is fired from Patrick Air Force Base toward the southeast, it will be over Africa in about 20 minutes. It's incredible how quickly an object moving 5 miles a second can get around.

Now I'll talk about the nature of the satellite. The Department of Defense has the problem of putting up a satellite and their intention is to be as absolutely sure of accomplishing this as they can. They plan to make six "earnest tries." Six times they will make earnest efforts to put a 21½-lb. satellite, or its brother, up into an orbit. We hope they will be successful with perhaps half of those tries. The first time the chances undoubtedly are the lowest—"bugs" of some sort or another tend to creep in and cause trouble. The second time the assurance will be higher and, as time goes on and more tries are made, we hope that as many as half of them will be successful. Then the IGY hopes to send up six more, making a dozen attempts all together; possibly no two of the satellites will be alike.

As I said, the Defense Department has to prove that they have put one up. How are they going to do this? A 20-in. sphere hundreds of

miles away is hard to see. They will put in each satellite a small transmitter operated by batteries, probably not solar batteries in the first ones, nor ordinary batteries, but super batteries lasting as long as possible. This small transmitter will probably operate on 108 megacycles per second (about 3 meters wave length) which will transmit at a phenomenally low power, measured in milliwatts. A very weak lamp uses 10 watts of current. Although the radio source is weak in the satellite, because they can't use very much energy from this finite battery source, the strength of the system lies in plans for very large antennas on the ground, which will capture minute signals. I've forgotten how many acres these antennas will cover, but they will be able to pick up the weak signal from the satellite every time the satellite comes within perhaps 600 miles, and not only determine that it came over, but measure where it was to an accuracy of the order of a minute or two of arc. That's the general precision with which they hope to observe the satellite by radio techniques. Furthermore, the same transmitter will, when requested from the ground, transmit information from a few measuring devices in the satellite. I'll go into that a little bit later.

Now, as you know, there are times when electronics fails. One trouble with satellites is that they're very hard to service. If something happens up there in the transmitter, there is not very much that can be done about it. This is where the Smithsonian Astrophysical Observatory comes into this picture. We have proposed to set up gound observing stations so that we can detect a satellite by its reflected sunlight. This has to be done with considerable care and at the right moment, because on the bright daylight side of the earth a satellite reflects sunlight but you can't see it and on the dark side the satellite will also be in the dark. The only time you can hope to see a satellite will be at the twilight period when the satellite is more or less overhead and the sun has already set a bit below the horizon so that the sky is not too bright. The observer, then, is in the shadow, the sky is rather dark, the satellite is up above in the sunshine. At that time a 20-in. sphere at a minimum distance of 200 miles will be just a hair brighter than the 6th magnitude which, as I'm sure most of you know, is the theoretical limit of the naked eye. What does that mean? It means that if you go out into beautifully clear country air with absolutely clear sky, no moon, no twilight, and you have extraordinarily good eyes—I'm sure I've never seen a 6th magnitude star—you will see the satellite with the naked eye, if you know where to look and when. When close, 200 miles up, it will move a little over a degree per second; that means across the moon's diameter in half a second. However, it doesn't take much optical aid—binoculars, for example—to make a 6th magnitude object really quite conspicuous.

Thus, a satellite should be readily observable if you know where

and when to look through binoculars. We hope to do it. We intend to set up two observing programs. I mention the visual one first because it's rather important. Suppose the radio transmitter in the satellite doesn't operate properly in the early stages. It's going to be touch and go to find out just what is happening to the satellite. Here it is sent off in an orbit. But the rocketry is not controlled in the last stage; it's sent off just like a 4th of July skyrocket, precisely built, however,—but it needn't go exactly in the direction one wishes. We hope the satellite launching will stay within about a degree or so of its proper course. Then, by the time it goes around once it can still be 60 miles or so away from the direction one expects to see it. So that the first few times around—if the transmitter is not functioning—there may be considerable question as to what happened to the satellite.

For that reason, the Astrophysical Observatory of the Smithsonian Institution has been assigned the task of organizing the optical programs. One important aspect of this task is the organization of the non-professional astronomers and other interested observers into groups so they can systematically look for the satellite and help us find out what happened to it in case the radio tracking did not function.

There is another time when amateur observations will be of extreme importance; that is when the satellite is about to come down. Although it goes up into the atmosphere above the earth where the air density is very low—considerably lower than in the vacuum of a thermos bottle—nevertheless, there's enough air up there to bring it down after a while. You've read various accounts in the newspapers of how long such a satellite will stay up and you've probably seen that these numbers didn't agree very well. There's a good reason for that because nobody knows. Even given an exact orbit, say 200 miles perigee minimum height, 800 miles maximum height, I don't think anybody on the planet can estimate the length of time it will stay up within an accuracy better than a factor of 3—and possibly not within a factor of 10 either way. We simply do not know the amount of air that exists at altitudes of 200 miles and above, to this degree of accuracy. My own estimate, and I don't have any reason to think it's going to be better than any others, is that the satellite will start falling rapidly in about a year. Now if that's 10 times too high, the satellite will start coming down in six weeks or so. If the air is 10 times less dense, then our best guess is that it may stay as long as 10 years.

So there is great uncertainty here. These astrophysical orders of magnitude sound fairly definite until you try to put them into time or dollars—then the uncertainty begins to show up. Uncertainties of only a factor of 2 become pretty important in dollars. In any case, as the satellite falls down into the atmosphere, the rate at which it comes down, or, in other words, the resistance of the atmosphere, is a direct measure of how much air there is up there. Simply by looking at an

object—in this case, a sphere with nothing in it at all except its mass,—just moving around in an orbit, one can determine the density of air at various heights up to approximately 300 miles. Rockets give us such data and results have been very surprising up to an altitude of a little more than 100 miles. We know that, at that altitude, the air density has fallen off by something like 1000 million from the sea level value. So the first result, the measure of the air density (air pressure), comes from simply watching the satellite. If we knew the composition of the air we could also tell the temperature. Both are extremely important data about the very high atmosphere.

I was leading to the point that in the last phases of this fall-in, the satellite is going to be changing its orbit very rapidly. First the computing machines will work on this problem, taking all the observations that have been made. The optical observations by the non-professionals, the radio observations that exist, the photographic observations, and any others that are made, will all be fed into one of these large mechanical brains which will predict where the satellite is going to be. This works out very well until the very last stages when the changes in the orbit are so fast that I don't think we can keep up with it. Here again the non-professional visual observer with his binoculars is of extreme value in the scientific aspects of the satellite program. As I said, the Astrophysical Observatory of the Smithsonian Institution has been assigned the task of initiating an optical observing program. Dr. J. Allen Hynek of Ohio State University has joined me in this project, heading it as Associate Director. For the non-professional astronomers, we have asked Dr. Armand Spitz, whom many of you know, to head the organization. If any of you would like to get into this program go to him or to your local astronomical organization—and volunteer to work with that group. Extremely valuable observations can be made by groups working in that fashion.

The problem of predicting where the satellite will be requires quite a performance from the computing machines. It takes considerable calculation just to predict where it will be at a given instant, such as at 12 o'clock noon, Greenwich civil time, on a given day. For ordinary astronomical predictions we calculate where a comet is going to be once every five days, which is usually quite adequate; a very fast moving comet requires a prediction every day. In the case of the satellite, predictions are required about once every minute, or 1440 times per day—and even then this is not a fine prediction series because in a minute the satellite moves 300 miles. We can locate points on the earth's surface 300 miles apart at minute intervals. Those computing machines had jolly well better compute an ephemeris position in less time than one minute, or we can't keep up with it.

I won't go on in that vein any more except to say that the motion of a satellite is quite complicated. The trouble is not that the sun

attracts it, for the sun's attraction is practically the same for the satellite as it is for the earth; the same holds for the moon. The trouble is that the earth is not a nice sphere with all the mass symmetrically placed about the center. Newton long since showed that if you had a sphere perfectly symmetrical about the center, the attraction would act as though the entire mass were concentrated at the central point. But the earth is in rotation so that it has an equatorial bulge; it's 13 miles greater in radius at the equator than it is at the poles. As a consequence, the earth has an extra "spare tire" attached around its equatorial belt—the sort of thing that happens to some of us as we pass into middle years! This extra mass around the equator attracts the satellite a little more strongly than it would, had all that mass been distributed spherically. This means that the orbital path, instead of remaining in a plane with the elongation of the ellipse in the same direction, keeps swinging around toward the west. It takes something like 40 or 50 days in the earlier stages, somewhat less time later on, for the pole or the plane to swing around in a westerly direction in this fashion. That's one complication. At the same time the long axis of the orbit will swing around in a forward sense as the satellite moves about the earth. Thus the nearest point to the surface of the earth moves in the plane around in the eastwardly or positive direction.

Then there are certain complicated little motions that take place so that the path at no time is a nice ellipse. Astronomers always talk about ellipses for the motions of heavenly bodies but there are no ellipses; they're always impossibly complex figures as is true in this case. One can handle the motion by complicated theoretical methods, a long series of expansions or by the computing machines. One of the little problems that worries us at the present time is the effect of what are known as gravitational anomalies on the earth. This may be a new term to some of you but the earth isn't even a perfect flattened-out spheroid. In some places there is a little more mass which attracts bodies like the satellite more, and attracts you too. I mean that if you're in a positive gravity anomaly you weigh a little more than if you were a few miles away where there is a negative anomaly. We haven't quite figured out yet what these effects are going to do to a satellite's motion. They may produce irregularities in the motion, which could make it hard for us to predict the motion as accurately as we would like.

Now a bit about the scientific results we expect from the satellite. The equipment inside each satellite will measure some physical quantity, probably two or three. In the earlier ones there may be devices that will measure, for example, the noise when a meteor pings the surface of the satellite. These little dust-like particles are not big enough to make visual shooting stars before they get into the earth's atmosphere (they are moving along anywhere from 7 miles a second up to 45 miles

a second), but, upon striking the surface of the satellite, they will make a noise which can be picked up by a very sensitive microphone. Some of this work was done by Dr. Lloyd Bohn at Temple University here a number of years ago with rockets and I hope that he is going to have a device in one of the early satellites to measure the number of these dust-type meteors that strike into the earth's upper atmosphere.

We would like to know how often a sphere like this will be punctured by meteors. I became interested in this just after the war in 1946 and made some calculations. I've been revising them ever since and I'm still not sure of the answer. The best way to get the answer is to put a little bit of air pressure in the satellite, then add a pressure measuring device, which will then give you a record of the pressure in the sealed up sphere. If a little hole is punctured by a meteorite, the pressure will fall off and by measuring the rate of fall you can determine the size of the hole made.

Then there will be devices for measuring the temperature, to see what the solar radiation does. You can also measure other physical characteristics of the satellite itself. That type of apparatus will almost certainly go in the first one and the measurements by this equipment will be sent to the ground by coded radio signals which will be picked up by the large antennas. Such a system is called a telemeter system. It has been used with great success in the sounding rockets in New Mexico and in other such investigations. Ten years ago it was a novel device. The Navy was instrumental in building early ones. Today it is an extremely reliable system whereby you could read a hundred instruments in a large rocket to an accuracy of one per cent, perhaps a hundred times a second, and collect all the information on the ground by radio. In the satellite system there will be only a few measuring devices and they won't be read so often as that because there won't be enough power. But a number can be read.

One observation which will certainly be made in one of the earliest satellites will be a measurement of cosmic rays. Down here, the atmosphere blocks off most of the cosmic rays but the cosmic rays in coming through cause secondaries; they hit atoms and break off more pieces that go on to strike us at high velocities. Cosmic rays are mostly hydrogen nuclei, moving with a velocity nearly that of light, but it is hard to distinguish the original ones that come from space from the secondaries they make in the atmosphere. This, incidentally, is one of the interesting points about the danger of cosmic rays in space. You would think that if you're out in space where you would be exposed to the primary cosmic rays, you had better put something between you and the cosmic rays. The effect of this would be very much like putting a window pane between you and a bullet. The bullet might not hit you if you didn't have the window pane there, but the shattered glass fragments might be more serious and damaging than the bullet

itself. Such is the effect of having a small amount of shielding from cosmic rays in space. You're worse off than if you didn't have any at all.

One of the major questions is not yet clearly resolved—whether some of the weaker cosmic rays come from the sun or whether their variations with solar activity are caused by changes in the earth's or the sun's magnetic field; thus variable magnetism could act as a valve, which controls the number that come down where we can measure them. If we can get direct cosmic ray measures out in the satellite orbit, well above the dense part of the earth's atmosphere, then we can learn a lot about the origin of the cosmic rays and perhaps settle this particular problem. At the same time we can measure the particles that are coming out from the sun. In recent years it has been demonstrated that the sun is shooting off hydrogen atoms,—not merely the nuclei as the electrons have to come along too, shooting off these hydrogen atoms up to 2000 miles a second and continuously. We used to think that space was completely clean, practically free of matter altogether. Now we believe that there are perhaps two hundred of these atoms at one time in any cubic centimeter-well beyond the earth's atmosphere. More shoot off when the sun becomes active (related to sunspot activities and so forth) to produce the Aurora Borealis when they strike the upper atmosphere.

And of course we are extremely interested in the solar activity in the far ultraviolet, in fact over all regions of the spectrum. Visual radiation comes in wave lengths of light about one 50-thousandth of an inch long. We can see through just a little crack in what is called the electro-magnetic spectrum, which starts 'way out in the gamma ray region, through the X-rays, the far ultraviolet, the near ultraviolet, the violet, the blue, green, yellow, red, deep red and finally infrared waves, before we come to the radio range. We can look through just a little crack or window in this entire range of energy, which is pouring on us from the sun and from the stars. The reason that this little window is so narrow is because of absorption by the earth's atmosphere. This atmosphere is very good for astronomers but very bad for astronomy. It is the greatest bane to astronomy that I know of. If we can get above the atmosphere, all this radiation comes pouring into our measuring instruments. The most interesting things happen to the sun out in the far ultraviolet where we can make no measures whatsoever from the earth's surface; from rockets, we can make measures of perhaps a minute's duration, but the satellite will stay in the sunlight 45 to 50 minutes at a time. We can then find out how the sun varies when it changes its light in the far ultraviolet and how it produces magnetic storms and the Aurora Borealis that I mentioned. The sun can produce trouble in power lines; it affects the ionosphere that reflects radio waves so that it can completely foul up

radio communication. We want to find out what is really going on in the sun.

Now the small satellite will probably not carry more than two or three of these experiments at a time, because most of the mass has to go into the radio equipment, the batteries, and the framework of the satellite. If you want to put an experiment into one of these, you are perfectly welcome to compete with the others—it will be a very real competition. All you have to do to your measuring equipment (essentially physical laboratory equipment) is to reduce the weight of everything—the equipment itself, plus batteries and all the connections—from a ton to 13 ounces, and you might win. That's the type of competition that's going on now and anybody who knows his way around in transistors and miniaturized radio sets has an advantage over the others in getting his equipment into this earth satellite. Just let Dr. James van Allen of Iowa State know of your desire and ability to work in this field. Other experiments will be done probably two at a time, such as the measurement of magnetic fields around the earth, other measurements of light, the composition of earth's atmosphere, and the amount of light scattered near the sun.

If we can ever get a camera up there to photograph the stars in the far ultraviolet, we are going to find that the sky looks altogether different from the way it looks now, photographically or to the naked eye. Mr. Robert J. Davis, a graduate student at Harvard, has calculated what the constellations would look like in the far ultraviolet, and he found that almost all of the bright UV stars are in the Milky Way, so the Milky Way will be very much more conspicuous than it is to the naked eye. The constellation Orion will be just a mass of stars. There will be a lot more in the belt; the only old friend you will miss is Betelgeuse. Betelgeuse is too red to have much light in the ultraviolet, so Orion will be cut off at his feet. On the other hand, the Big Dipper will be reduced to two conspicuous stars, so it can't hold water any more. It will be a very interesting day when you can see actual photographs made in this far ultraviolet X-ray region, reaches of the spectrum that never have been seen or photographed by men.

With reference to this satellite, I'd like to mention some of the results that we can hope to get by optical means and to discuss the program that we are initiating at the Astrophysical Observatory of the Smithsonian Institution. We hope to set up about 12 stations over the world utilizing large Schmidt cameras with about 20-in. aperture and about 20-in. focal length with a 30-in. mirror. In the stations we hope to measure the time accurately to about a 1000th of a second, keeping a crystal clock tuned to WWV and then measuring the instants of the photographs. I won't go into details of that except to say that we believe we can measure across the direction of the path to

an accuracy of about 2 seconds of arc. That would be about 1000th of the sun's or the moon's diameter. Along the direction of motion the accuracy should be of the order of 5 seconds of arc.

The significance of this becomes apparent. Suppose at a given moment that we know, from all the observations, the orbital path of the satellite to a high degree of accuracy. Then when we know the direction to a point in the satellite's orbit at a precisely determined time, we can determine our position on the earth to an accuracy of a few feet. In practice we will keep improving our knowledge of both a satellite's orbit and the positions of the observing stations. We hope indeed by this method to be able to determine the relative positions of about 12 to 15 stations over the earth, with respect to each other and with respect to the center of the earth, to an accuracy of something like 30 feet out of the 4000 miles to the center of the earth. That is about 10 times as accurate as measurements by geodetic surveys and we hope then to be able to tie in the surveys made in the United States with those made in other parts of the world. Astronomically we can't yet tie those together with an accuracy even as high, perhaps, as 300 feet. Some islands are off by as much as 2000 feet or more by astronomical measures. And then we can determine the shape of the earth. We can determine the size of the equatorial bulge and find out whether the earth is longer in one direction around the equator than it is in another direction, to see whether there is a third axis of the earth. Then we can determine quantities concerning the gravity, the density of material, and isostasy, the plastic flow represented by the way continents spring back up after being depressed by ice in the Ice Ages. You perhaps know that Finland is still rising That's the type of result that we hope to get from these precise observations.

I've been asked to report on the day's proceedings. Homer E. Newell of the Naval Research Laboratory opened the morning session with a talk about Project Vanguard and the International Geophysical Year. That's a long story involving some 43 nations. The huge program includes antarctic studies, oceanographic studies, studies of the aurora and so forth, and the satellite program is blended into that, as I mentioned earlier.

Then Krafft A. Ehricke, of Convair, discussed aeronautical and space medical research with automatic satellites That's looking forward to the day when men ride out into space. There are extremely important problems of the satellite situation that already exist in our aircraft; men now go far higher than they are able to exist unless they take an environment along with them—that is, atmosphere, pressure, moisture, and so forth. Studies of that sort can then very possibly be made by the satellite program, for future satellite programs.

Next George H. Clement of the Rand Corporation discussed a possible Moon Rocket, and some of the problems, including how you

shoot a body out there and how you make it either land on the moon or go around the moon so you can photograph the other side. One of the very interesting problems of the moon is whether it has any atmosphere at all. Out in space, we could determine whether there is a little bit of residual atmosphere—maybe some heavy gas like krypton or xenon. We know now that there is less than about 10^{-12}, which is one over a million millions as much air on the moon as there is above the earth's surface. So the moon rocket looks like a development that will follow very shortly after the first satellites. One of the interesting results will be to settle the question of whether maria, or so-called seas of the moon, are old lava flows and hard, or whether (as Thomas Gold in England suggests) they may be just big bowls filled with dust. He warns all travelers to the moon not to set their rockets down in the middle of one of the maria because they may settle into a mile deep ocean of dust. This would be very undesirable! We should know about this question before we send any manned vehicles up there.

In the afternoon program, the first lecture was by Lyman Spitzer, Jr., of the Princeton Observatory, who was discussing astrophysical research with an artificial satellite. He stressed particularly the problem that I mentioned, of measuring the earth's density and pressure in the high atmosphere, how much air is there, and then went into the problem of putting telescopes into satellites. This is a bit farther into the future, because the present $21\frac{1}{2}$-lb. sphere, with a 1-lb. allowance for an experiment, will not allow much of a telescope. But later on we expect to have telescopes there that can be directed. We already have them on the rockets and that's a much more difficult task because you have so little time to get the telescope pointed towards the sun. With the satellite, then, we shall be able to measure the individual stars in the far ultraviolet; a great many results of scientific interest will come out of this.

Just to give you an example, we're very much interested in the radiation of the night sky. When you look at the night sky, you see that there's a certain amount of light. I'm not talking about the artificial light you get over a city. At this stage of the game, with very simple equipment, one can hope to distinguish among four different sources of night sky light: light from the upper atmosphere, light from the solar system, light from the Milky Way and light from the distant galaxies. It may also be possible with rather simple equipment, to separate these sources in the far ultraviolet and in the near ultraviolet. Even in the visual region we could possibly determine some answers about cosmology. Are we living in a universe in which everything is blowing up and all the galaxies moving away? Are galaxies short-lived or, as some of the British suggest, would the system always look the same whether we see it now or ten billion years in the past, or ten billion years in the future? We may be able to dis-

criminate among some of those theories simply by measuring the amount of light in the background of the skyline, down to 5 or 10 per cent level of the night sky brightness, the part that comes from the very distant galaxies at incredible distances into space.

Next we heard a discussion by John de Nike of The Glenn L. Martin Company on the effect of the earth's oblateness and atmosphere on the satellite orbit. I think I can summarize his results very quickly: they are complex!

Then Lovell Lawrence of Chrysler Corporation discussed what he called the ASTRO System, the Artificial Satellite Time and Radio Orbit System. He needs *three* artificial satellites. If you had a ship at sea in cloudy weather, by means of a radio antenna and a complicated system you would be able to determine your position at sea, when one of three satellites came over. Even though impracticable yet, this indicates, I think, some of the potential future uses of satellites.

Then Richard W. Porter, who is the head of the Technical Panel for the Earth Satellite Program, discussed the very difficult and also very important problem of whether it is possible to recover physical information from a satellite after it falls through the atmosphere. In rocketry this turned out to be fairly simple. One of the first ones sent up—a V-2 rocket—had a camera in it in a very strong cassette. Pictures were taken from the top of the trajectory and then the rocket came down again. It was going so fast when it struck the surface of the earth that it made a hole about 35 ft. deep and 35 ft. across. Out of several tons of material they recovered perhaps 200 pounds, not including the cassette. The solution of this problem was surprisingly simple. The difficulty was that this rocket was streamlined. It's a good idea to have a rocket streamlined when it's going up, but if it is to come down slowly, it should not be streamlined. The simplest way to solve this is to blow it in two in the middle. This was done during descent and the pieces came down aerodynamically "dirty," as Dr. Porter calls it. As a consequence they landed at moderately reasonable speeds so that the film could be recovered. The problem is a little more complicated with the satellite because it has to be kept from burning up on the way down.

Just a word or two about the future beyond this very first step that's being made. The satellites can be divided into classes by weight, starting off by say a tenth of this one that is planned—a 2-lb. satellite, which is quite worthless. I can't see any merit in sending one up because you can never prove that it has been done and it has no scientific value. When the order of about 10 or 20 pounds is reached, a satellite object can be built which can, at least, be observed and instrumented to some extent. The next step, using more powerful rockets and with mastery of the technique, is to send up something about 10 times as heavy, say 200 pounds. At this stage, perhaps nuclear

power supplies can be utilized, so that the power system will last for years, even decades or centuries. Or perhaps solar batteries can be used to transform solar radiation into electrical energy. Probably a transistorized television system can be included to take photographs from such a satellite.

The next step is perhaps the 2000-lb. satellite, in which all of these things can be done better through the inclusion of a small physical laboratory. At about this size we come, not to the manned rocket, but what I like to think of as the puppet-type rocket. A puppet, remotely controlled from the earth by radio techniques, can do any of the useful things up there that man can do. You see, the senses are useless in space, except for sight and touch and, as a consequence, the puppet needs only television eyes and a feedback servomechanism. Then you can, in effect, be in the rocket by remote control, controlling a puppet, not by strength but by radio, to do about anything a man can do. This appeals to me because it has not been demonstrated how man is going to react to zero gravity fields, which will prevail in space. The puppet would probaby be happier working in a vacuum, so there would be no need to carry any air up there. I think he would be more effective than a man. He could be shut off for a year or two and then put to work again later on if there were some need for it.

I do want to mention a type of scientific result that I think would be extremely valuable to all of us when even the 200-lb. satellite perhaps becomes possible—research on meteorology and weather conditions. I think this progress will complete the revolution of meteorology, which is now under way in the form of mechanizing the collection of data and the calculations of weather forecasts. We have almost reached the point where a machine can do better than a man in forecasting weather. I won't elaborate, but merely note that three-fourths of the earth's surface is covered with water; almost nothing is known about the cloud cover over the oceans, and only a very small amount over the land. A relatively simple satellite—by television camera—can photograph the earth continuously, covering the entire surface more frequently than once a day.

I think the greatest scientific discoveries resulting from the satellites will be those we haven't anticipated. Science has always surprised us this way.

To me the satellite represents the most marvelous scientific development that has happened during my life.

High-Speed Printer.—The General Electric Company has developed a special-purpose, high-speed printer capable of turning out more than 2500 lines a minute. Additional developmental work could step up the speed of the printer to "several hundred lines a second," the company added.

The revolutionary electro-magnetic printer—about the size of a shoebox—has been used to print results of numerical calculations by fast analog computers.

The principle of the new printer could be applied to printing such items as paychecks or utility bills from lightning-fast digital computers, they added.

The process, called ferromagnetography, was described by Robert D. McComb of Schenectady, a G-E engineer, in a talk before the Rochester section of the Technical Division, Photographic Society of America.

McComb said a similar electromagnetic technique has been applied to "facsimile" reproduction of pictures, line drawings and other irregular-shaped lines or characters.

One such printer and the application of ferromagnetography were developed in G-E's General Engineering Laboratory in Schenectady, where pioneer work in radio, TV, electronics and other technical fields has been done through the years.

McComb said the printer could eliminate a troublesome bottleneck in transmitting data from speedy computers.

"Computers can run at fantastically high speeds, but the results of their computations must be recorded before their human masters can use the data," McComb said.

"Lack of a satisfactory outscriber—the mechanism that writes out data from the computer—has been a limiting factor in the speed and efficiency of the computer, or else has complicated the computer output data system and computer operation."

Present-day tabulating machines operate at about 150 lines a minute, with some of the newest ranging as high as 900 lines a minute.

McComb added that he foresaw no immediate threat to the modern teletype or printing press "because the applications are so different."

The ferromagnetic printing mechanism is contained in a metal box only 6½ in. wide, 5½ in. high and 15 in. deep. Inside, McComb said, is the equivalent of a linotype machine, printing press and all accessory equipment needed for printing.

The ferromagnetographic process makes a complete cycle during each revolution of a magnetic cylinder. The basic principle involves these steps:

1. An image of the lines to be printed—numeral, letter, drawing, etc.—is formed by magnetizing the outlines on the rotating magnetic cylinder.

2. Tiny ferromagnetic particles—"iron filings"—stick to the magnetized area. These particles are black in color and become, in effect, the "ink."

3. Upon contact with a roll of treated paper—heated and containing a special wax—pressed against the magnetized cylinder, the metal particles are transferred to the paper. As the paper cools, the particles adhere to the paper and are absorbed in it.

These steps, McComb pointed out, "are analogous to plate-making, inking and printing in the graphic arts."

OSCULATORY EXTRAPOLATION AND A NEW METHOD FOR THE NUMERICAL INTEGRATION OF DIFFERENTIAL EQUATIONS

BY

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REAL FUNCTIONS

For $f \equiv f(x)$ tabulated at equally spaced points x_k, $f_k \equiv f(x_k)$, the familiar binomial coefficients $\binom{n}{m}$ are employed in ordinary extrapolation and checking[2] according to the formula

$$f_{n+1} = \sum_{m=1}^{n} (-1)^{m-1} \binom{n}{m} f_{n-m+1} + \Delta^n f_1, \tag{1}$$

where the remainder $\Delta^n f_1$ vanishes when f is a polynomial of the $(n-1)^{\text{th}}$ degree. Now one may find also "confluent binomial coefficients" or multipliers of both f_k and f'_k to be utilized in formulas similar to (1).

These osculatory extrapolation formulas are extremely accurate, being exact for f any polynomial of the $(2n-1)^{\text{th}}$ degree and otherwise having a remainder term that is approximately proportional to $\Delta^{2n} f$, the factor of proportionality being very much less than 1 and becoming progressively smaller as n increases (unlike the comparatively large and unchanging factor of 1 for the $\Delta^n f_1$ in (1)).

To conform with standard notation, h denotes the tabular interval of $f(x)$, $x_k \equiv x_0 + kh$, $f_k \equiv f(x_0 + kh)$, $f'_k \equiv f'(x_0 + kh)$, and the n subscripts k of f_k and f'_k will range from $-[(n-1)/2]$ to $[n/2]$, $[u]$ denoting the largest integer not exceeding u. Then the osculatory extrapolation formulas are obtained from the osculatory interpolation formula for f_p in (1),[3] by setting $p = [n/2] + 1$ to get

$$f_{[n/2]+1} = \sum_{k=-[(n-1)/2]}^{[n/2]} (C_k f_k + h D_k f'_k) + R_{2n}, \tag{2}$$

where

$$R_{2n} = f^{(2n)}(\xi) h^{2n} \cdot F_n, \quad x_{-[(n-1)/2]} \leq \xi \leq x_{[n/2]}, \tag{3}$$

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[2] Throughout this article there are references to techniques of interpolation, differencing, and numerical integration which might not be all clear to a reader unfamiliar with numerical analysis. In such cases, the reader is referred to either of these two standard references: J. B. SCARBOROUGH, "Numerical Mathematical Analysis," Baltimore, Johns Hopkins Press, 1950; W. E. MILNE, "Numerical Calculus," Princeton, Princeton University Press, 1949.

[3] The boldface numbers in parentheses refer to the references appended to this paper.

TABLE I.—*Table of C_k and D_k, with F_n.*

2-point

$C_0 = 5 \quad D_0 = 2$
$C_1 = -4 \quad D_1 = 4$
$F_2 = \frac{1}{6}$

3-point

$C_{-1} = 10 \quad D_{-1} = 3$
$C_0 = 9 \quad D_0 = 18$
$C_1 = -18 \quad D_1 = 9$
$F_3 = \frac{1}{20}$

4-point

$C_{-1} = 47/3 \quad D_{-1} = 4$
$C_0 = 64 \quad D_0 = 48$
$C_1 = -36 \quad D_1 = 72$
$C_2 = -128/3 \quad D_2 = 16$
$F_4 = \frac{1}{70}$

5-point

$C_{-2} = 131/6 \quad D_{-2} = 5$
$C_{-1} = 575/3 \quad D_{-1} = 100$
$C_0 = 100 \quad D_0 = 300$
$C_1 = -700/3 \quad D_1 = 200$
$C_2 = -475/6 \quad D_2 = 25$
$F_5 = \frac{1}{252}$

6-point

$C_{-2} = 142/5 \quad D_{-2} = 6$
$C_{-1} = 426 \quad D_{-1} = 180$
$C_0 = 825 \quad D_0 = 900$
$C_1 = -400 \quad D_1 = 1200$
$C_2 = -750 \quad D_2 = 450$
$C_3 = -642/5 \quad D_3 = 36$
$F_6 = \frac{1}{924}$

7-point

$C_{-3} = 353/10 \quad D_{-3} = 7$
$C_{-2} = 4018/5 \quad D_{-2} = 294$
$C_{-1} = 6027/2 \quad D_{-1} = 2205$
$C_0 = 1225 \quad D_0 = 4900$
$C_1 = -6125/2 \quad D_1 = 3675$
$C_2 = -9114/5 \quad D_2 = 882$
$C_3 = -1911/10 \quad D_3 = 49$
$F_7 = \frac{1}{3432}$

8-point

$C_{-3} = 1487/35 \quad D_{-3} = 8$
$C_{-2} = 6816/5 \quad D_{-2} = 448$
$C_{-1} = 40768/5 \quad D_{-1} = 4704$
$C_0 = 10976 \quad D_0 = 15680$
$C_1 = -4900 \quad D_1 = 19600$
$C_2 = -58016/5 \quad D_2 = 9408$
$C_3 = -18816/5 \quad D_3 = 1568$
$C_4 = -9376/35 \quad D_4 = 64$
$F_8 = \frac{1}{12870}$

9-point

$C_{-4} = 6989/140 \quad D_{-4} = 9$
$C_{-3} = 75087/35 \quad D_{-3} = 648$
$C_{-2} = 92664/5 \quad D_{-2} = 9072$
$C_{-1} = 225792/5 \quad D_{-1} = 42336$
$C_0 = 15876 \quad D_0 = 79380$
$C_1 = -206388/5 \quad D_1 = 63504$
$C_2 = -165816/5 \quad D_2 = 21168$
$C_3 = -243648/35 \quad D_3 = 2592$
$C_4 = -50301/140 \quad D_4 = 81$
$F_9 = \frac{1}{48620}$

10-point

$C_{-4} = 7255/126 \quad D_{-4} = 10$
$C_{-3} = 22345/7 \quad D_{-3} = 900$
$C_{-2} = 262035/7 \quad D_{-2} = 16200$
$C_{-1} = 138720 \quad D_{-1} = 100800$
$C_0 = 149940 \quad D_0 = 264600$
$C_1 = -63504 \quad D_1 = 317520$
$C_2 = -173460 \quad D_2 = 176400$
$C_3 = -560160/7 \quad D_3 = 43200$
$C_4 = -166455/14 \quad D_4 = 4050$
$C_5 = -29345/63 \quad D_5 = 100$
$F_{10} = \frac{1}{184756}$

11-point

$C_{-5} = 82451/1260 \quad D_{-5} = 11$
$C_{-4} = 572935/126 \quad D_{-4} = 1210$
$C_{-3} = 1941445/28 \quad D_{-3} = 27225$
$C_{-2} = 2506515/7 \quad D_{-2} = 217800$
$C_{-1} = 667920 \quad D_{-1} = 762300$
$C_0 = 213444 \quad D_0 = 1280664$
$C_1 = -569184 \quad D_1 = 1067220$
$C_2 = -3869580/7 \quad D_2 = 435600$
$C_3 = -4807935/28 \quad D_3 = 81675$
$C_4 = -2407295/126 \quad D_4 = 6050$
$C_5 = -740641/1260 \quad D_5 = 121$
$F_{11} = \frac{1}{705432}$

which is expressible in the more convenient approximate form

$$R_{2n} \cong \Delta^{2n} f \cdot F_n, \tag{3'}$$

and where in both (3) and (3') F_n is the remainder coefficient (given in (1)) for $p = [n/2] + 1$, namely $(n!)^2/(2n)!$. The quantities C_k and $D_k, k = -[(n-1)/2]$ to $[n/2]$, were calculated exactly for $n = 2(1)11$, using the procedure which is explained in (1) for computing osculatory interpolation coefficients, for $p = [n/2] + 1$,[4] and they are given in Table I, together with F_n, the coefficient of $\Delta^{2n} f$ in (3'):

Formula 2 has many obvious applications when both f and f' are tabulated together:

I. Thus, as already mentioned in the first paragraph, it is a much more accurate extrapolation formula when compared with the ordinary extrapolation formula even for $2n$ points, as seen at once from the tabulated values of F_n.

II. Then (2) (after transposing $f_{[n/2]+1}$ to the right side) can be used to check tables for errors, just like differencing, but with the following two advantages over the latter:

(*a*) Theoretically the total number of quantities f and f' which are required for checking at intervals of h is less than the total number of quantities f alone at intervals of h, because having both f and f' is in a certain sense equivalent to f at an average interval of $h/2$ instead of h. As a result not only will there be fewer terms to add together, but usually each term will be the product of f_k or hf'_k by a smaller number. Roughly speaking, the advantage is like that in the requirement of lower order differences in checking when the tabular interval is halved.

(*b*) Also, since the coefficients C_k and D_k for n confluent points have the same order of magnitude as the ordinary extrapolation coefficients for $2n$ points, there is an added advantage in that the D_k are multiplied by the h which is usually much less than 1. This reduces the error due to round-off of f'_k in the use of (2), so that the effect of errors in f_k might be revealed more clearly, with greater ease in their location.

III. For the numerical integration of differential equations of the form $y' = \phi(x, y)$, formula 2, with y and y' replacing f and f', enables one to find from several initial values of y_k and $y'_k = \phi(x_k, y_k)$, $k = -[(n-1)/2]$ to $[n/2]$, the next value of y_k, say $y_{[n/2]+1}$. Then the corresponding value of $y'_{[n/2]+1}$ follows from $y' = \phi(x, y)$. Thus suppose that the next y_k is attainable with essentially the same accuracy as the preceding values of y_k, as determined by estimation of the remainder terms R_{2n} in (2) (putting aside for the moment consideration of errors caused by the round-off in the initial quantities). Then one sees the great usefulness

[4] In the computation of C_k and D_k, the author was checked by Miss Isabelle Arsham.

of these osculatory extrapolation coefficients as the basis of a very simple and direct procedure for the stepwise numerical integration of $y' = \phi(x, y)$ by continuing this process. No differences are needed here. Continuation of this process to successive values of y_k and y'_k is not an operation like "lifting by the bootstraps" because we are not simply basing everything upon the initial values of y_k and y'_k and merely extrapolating indefinitely, since at each stage we are introducing a new quantity by using the equation $y' = \phi(x, y)$.

But the chief advantage in the use of (2) for numerical integration of $y' = \phi(x, y)$ over the standard procedures such as Adams-Bashforth, Runge-Kutta, etc., is that the latter all seem to be based upon a polynomial approximation, either Lagrangian or Taylor series, for either y' or y, and in a sense they are inherently of the order of n-point accuracy; whereas here the basis is the $2n$-point Hermite osculatory interpolation formula which employs both y and y' (twice as much information within the same range of the argument x). For example, the basis of the Adams-Bashforth method is an extrapolation from a formula obtained by integration of the n-point polynomial approximation for only the y', and the derivation makes no use of the available information in all the known values of y. Examination of remainder terms in Adams-Bashforth and other similar methods shows them to be something like a sizeable fraction of an n^{th} difference, whereas those here are extremely small by comparison, $\sim F_n \Delta^{2n} y$, which allows the choice of a much larger value of h with consequently fewer total steps and the saving of computation time and labor. It is likely that in the vast literature containing so many different formulas for numerical integration, somebody might have already given formulas for integration that follow from taking the lowest values of n in (2); but the author is unaware of any systematic series of formulas found by applying Hermite's osculatory interpolation formula to extrapolation.[5]

However, (2) by itself is not sufficient for most practical problems in stepwise integration due to the increasing size of the coefficients C_k and D_k as n increases. Even though the theoretical remainder term R_{2n} in (2) is extremely small, in actual use y_k and y'_k are known only approximately, and their round-off errors are multiplied by coefficients that become rather large with n. In an example where (2) is employed for $n = 5$, after not too many steps the resulting y_k and y'_k might be unusably inaccurate if there were few extra places in the initial values of y_k.[6]

[5] The editors indicated that there was overlapping, to a certain extent, of this idea for solving $y' = \phi(x, y)$ with the work of P. C. Hammer and his associates at the University of Wisconsin, citing P. C. HAMMER and J. W. HOLLINGSWORTH, "Trapezoidal Methods of Approximating Solutions of Differential Equations," *Math. Tables and Other Aids to Comput.*, Vol. IX, pp. 92–96 (1955).

[6] Of course this same problem exists ni the usual familiar procedures for integration employing $\Delta y'_k$, $\Delta^2 y'_k$, etc., if those differences are computed from rounded values of y'_k, so that there would not be much, if anything at all, gained by expressing (2) in terms of differences of y_k and y'_k.

But if we confine ourselves to the use of (2) for values of $n \leq 5$ (and it is hard to conceive of a practical numerical integration problem where the size of h requires the use of differences up to $\Delta^{10}y$), the situation can be remedied by the derivation of a series of entirely new type of integration formulas that are midway between osculatory interpolation and osculatory extrapolation, and which are capable of refining the values of $y_{[n/2]+1}$ obtained by (2).

Offhand there appear to be two new types of "$n\frac{1}{2}$-point" (that is, of accuracy comparable to $(2n + 1)$-point interpolation) osculatory extrapolation formulas that can be derived. The simpler one is based upon the function and its derivative at n points and the function alone at the $(n + 1)^{\text{th}}$. It can be derived from the general divided difference formulas by taking confluent divided differences, but leaving the first or last, that is, the $(2n + 1)^{\text{th}}$ point distinct, and its remainder is also well-known. But such formulas do not fit into the scheme of refining the results of (2) in the stepwise numerical integration of $y' = \phi(x, y)$ as the more unusual type of formula (not even a confluent form of anything previously known) based upon the function and its derivative at n points and the *derivative alone* at the very next or $(n + 1)^{\text{th}}$ point, which we use to evaluate the function at that same $(n + 1)^{\text{th}}$ point. Such an osculatory formula, strictly speaking, is neither purely extrapolatory because we do have some information in the value of the derivative at that $(n + 1)^{\text{th}}$ point, nor is it purely interpolatory because the unknown functional value is for a point outside the interval containing the arguments where the function is known. Although an explicit formula for the remainder term seems somewhat elusive because the Rolle's theorem type of argument appears inapplicable, in all likelihood such an $n\frac{1}{2}$-point osculatory formula has a remainder term that is less than the $\Delta^{2n}f \cdot F_n$ for the n-point osculatory formula. Also, in comparison with the C_k and D_k in (2), its coefficients are considerably smaller, as might be expected from a formula that is more interpolatory in nature than (2).

This latter type of $n\frac{1}{2}$-point osculatory formula is ideally suited as a refining formula to be used in conjunction with (2) in the numerical integration of $y' = \phi(x, y)$. Thus after we find $y_{[n/2]+1}$ from (2) and thence $y'_{[n/2]+1}$ from the differential equation, a noticeable amount of accuracy is lost even in one step when n is larger, especially $n = 4$ or $n = 5$. But now the use of a refining formula for $y_{[n/2]+1} (\equiv f_{[n/2]+1})$ like

$$f_{[n/2]+1} \cong \sum_{k=-[(n-1)/2]}^{[n/2]} A_k f_k + h \sum_{k=-[(n-1)/2]}^{[n/2]+1} B_k f'_k, \tag{4}$$

would be expected to yield greater accuracy because it is more interpolatory and it is more like a $(2n + 1)$-point rather than a $2n$-point

confluent formula. But most important and crucial is the much smaller size of the coefficients A_k and B_k when compared with C_k and D_k in (2), with particular emphasis upon the fact that the lesser accurate $f'_{[n/2]+1}$ ($\equiv y'_{[n/2]+1}$), the only new quantity in (4), is multiplied by $hB_{[n/2]+1}$, where h is usually a small fraction of 1, and $B_{[n/2]+1}$ is around $\frac{1}{4}$ for $3 \leq n \leq 5$. Thus use of (2) for n points followed by (4) for $n\frac{1}{2}$ points should yield a refined value for $y_{[n/2]+1}$, and thence $y'_{[n/2]+1}$, which would about restore their accuracy so that this same process, (2), followed by (4), can be repeated upon the next set of n consecutive pairs f_k and f'_k. Any doubt as to the adequacy of (4) as a refining formula can be resolved before proceeding to the next stage, by employing it for a second time with the newer and more accurate $f'_{[n/2]+1}$ to make sure that there is no further significant variation in $f_{[n/2]+1}$ (more simply, to ascertain that $hB_{[n/2]+1}\{\phi(x_{[n/2]+1}, y_{[n/2]+1}$ from (2)) $- \phi(x_{[n/2]+1}, y_{[n/2]+1}$ from (4))$\}$ is negligible.

The coefficients A_k and B_k in (4) were calculated exactly for $n = 1$ to 5 by solving the sets of $2n + 1$ linear equations formed by taking the function $f(x)$ to be x^j, $j = 0, 1, \cdots, 2n$, at $x = -[(n-1)/2]$ to $[n/2]$ for $f(x)$ and to $[n/2] + 1$ for $f'(x)$, at intervals of $h = 1$. The correctness of every n-point and $n\frac{1}{2}$-point formula derived for $n \leq 5$ was checked functionally by examples where f was chosen to be polynomials of every degree $\leq 2n - 1$ or $\leq 2n$, respectively. The results for $n = 1$ and 2 are the trapezoidal and Simpson's rule, respectively, and are not new, being included below only for the sake of completeness. But the formulas for $n = 3$, 4 and 5 are apparently entirely new and are exact for $f(x)$ a polynomial of maximum degree 6, 8, and 10, respectively.

The derivation of the formulas (4) beyond $n = 5$ seemed hardly worthwhile because $n = 5$ yields already the equivalent of a 10^{th} difference or 11-point approximation, all compressed into a total range of $5h$ instead of $10h$ (which should make it effectively around 1000 times as accurate as any 10^{th} difference formula at intervals of h). Another reason for not going further is the increasing size of the coefficients of f_k and f'_k. For example, consider in $n = 5$, $A_0 = -A_1 = 2000/137$, so that if the first approximations to f_3 and f'_3 are not perfect, even though $hB_3 = 30h/137$ may be a small multiplier for f'_3 in (4), the 2000/137 will be responsible for an appreciable contribution to the error in f_3 through magnification of the error in the $f_0 - f_1$. Still, comparing the $5\frac{1}{2}$-point formula with the 5-point formula, the coefficients A_k and B_k are only around 10 per cent of the C_k and D_k.

In actual numerical integration problems the user may find formulas (2) and (4), for $n = 3$ or 4, of much more practical value than for $n = 5$, owing to the considerably smaller values of C_k and A_k, even though the remainder terms for $n = 5$ are considerably less. In fact, the user who tries this procedure for the first time might concentrate on using first only $n = 3$, since it is already considerably more accurate and conveni-

ent than most integration formulas in use, and then he might try $n = 4$ for either much greater accuracy or for allowing a larger interval h.

The following schedule presents the extrapolation formulas ("n-point" osculatory) with the corresponding refining formulas ("$n\frac{1}{2}$-point" osculatory) in a form convenient for immediate use, for $n = 1$ (1) 5:

$$1\text{-point}: f_1 = f_0 + hf'_0$$

$$1\tfrac{1}{2}\text{-point}: f_1 = f_0 + \frac{h}{2}(f'_0 + f'_1)$$

$$2\text{-point}: f_2 = 5f_0 - 4f_1 + 2h\{f'_0 + 2f'_1\}$$

$$2\tfrac{1}{2}\text{-point}: f_2 = f_0 + \frac{h}{3}\{f'_0 + 4f'_1 + f'_2\}$$

$$3\text{-point}: f_2 = 10f_{-1} + 9f_0 - 18f_1 + 3h\{f'_{-1} + 6f'_0 + 3f'_1\}$$

$$3\tfrac{1}{2}\text{-point}: f_2 = f_{-1} + \frac{27}{11}(f_0 - f_1) + \frac{3h}{11}\{(f'_{-1} + f'_2) + 9(f'_0 + f'_1)\}$$

$$4\text{-point}: f_3 = \frac{47}{3}f_{-1} + 64f_0 - 36f_1 - \frac{128}{3}f_2 + 4h\{f'_{-1} + 12f'_0 + 18f'_1 + 4f'_2\}$$

$$4\tfrac{1}{2}\text{-point}: f_3 = f_{-1} + \frac{32}{5}(f_0 - f_2) + \frac{6h}{25}\{(f'_{-1} + f'_3) + 16(f'_0 + f'_2) + 36f'_1\}$$

$$5\text{-point}: f_3 = \frac{131}{6}f_{-2} + \frac{575}{3}f_{-1} + 100f_0 - \frac{700}{3}f_1 - \frac{475}{6}f_2 + 5h\{f'_{-2} + 20f'_{-1} + 60f'_0 + 40f'_1 + 5f'_2\}$$

$$5\tfrac{1}{2}\text{-point}: f_3 = f_{-2} + \frac{1}{137}[1625(f_{-1} - f_2) + 2000(f_0 - f_1) + 30h\{(f'_{-2} + f'_3) + 25(f'_{-1} + f'_2) + 100(f'_0 + f'_1)\}]$$

COMPLEX FUNCTIONS

The method of numerical integration described above for real functions may be modified to give a newer, more compact, and (in certain respects only) more accurate procedure for the stepwise numerical integration, over a Cartesian grid of length h, of differential equations of the form $w' = \phi(z, w)$, in a region of the complex plane where $w(z)$ is an analytic function. In a previous paper (2) the author gave a method for stepwise integration based upon the integration of a Lagrangian interpolation polynomial over a number of points of the grid, employing functional values alone. This present method is based upon the Hermite osculatory interpolation formula which is used to extrapolate for functional values just outside a single square of the grid. The coefficients in these new formulas tend to be somewhat larger than those used in (2), so that errors due to approximate entries may tend to propagate at a faster rate. But final judgment on that point must be reserved until an adequate number of examples are done by both methods, because the saving in the total number of computational steps in this newer method might still reduce the propagated rounding error to below that in (2). On the other hand, the integration formulas are more accurate because the approximation is more condensed and not spread out over more than a single square for the purely extrapolatory part and still

giving 7th degree accuracy in $w(z)$,[7] whereas in the preceding paper (2) the 7th or 8th degree approximations were spread out over four squares.

Instead of systematically tabulating all types of extrapolation formulas, with refining formulas to match them, for the former the choice was made of the very symmetric-looking 7th degree approximation based upon the tabulated values of $w(z)$ and $w'(z)$ at each of the corners of a single square. Since we assume that the rather unwieldy looking complex integral form of the remainder term (known now for the osculatory extrapolation formulas only) is to be negligible as a result of such a high degree approximation, the = sign will be used below even though it really denotes an approximate equality. Let z_k denote the grid points $z_0 + kh$, where k is a small complex integer, and suppose that we know initially $w_k \equiv w(z_k)$ and, of course, $w'_k \equiv w'(z_k)$ from $w' = \phi(z, w)$, at the grid points z_0, z_1, z_i and z_{1+i}. Consider the configuration of dot and cross points which is given below:

×	×	
z_{2i}	z_{1+2i}	
•	•	×
z_i	z_{1+i}	z_{2+i}
•	•	×
z_0	z_1	z_2

If from the knowledge of $w(z_k)$ and $w'(z_k)$ at z_0, z_1, z_i, and z_{1+i} we can determine $w(z_k)$ and $w'(z_k)$ at z_2, z_{2+i}, z_{1+2i} and z_{2i}, the initial configuration is repeated a step both to the right and above, so that we can proceed over the entire grid, row by row or column by column. To calculate $w_j \equiv w(z_j)$, $j \equiv 2, 2 + i, 1 + 2i$, or $2i$, one has the following series of formulas which is analogous to (2) above:

$$f_j = \sum_k (C_k^{(j)} f_k + h D_k^{(j)} f'_k), \tag{5}$$

where $\sum_k$ is for $k = 0, 1, i$ and $1 + i$.

But here again the large size of the coefficients $C_k^{(j)}$ and $D_k^{(j)}$ in (5) makes it desirable to have some series of refining formulas which would be analogous to (4) above, for $j = 2, 2 + i, 1 + 2i$ and $2i$, and the most fitting appear to be of the following type, of 8th degree accuracy:

$$f_j = \sum_k A_k^{(j)} f_k + h \sum'_k B_k^{(j)} f'_k, \tag{6}$$

where the $\sum_k$ is still for $k = 0, 1, i$ and $1 + i$, but the $\sum'_k$ is for $k = 0, 1, i, 1 + i$ and also j.

[7] The 8th degree refining formulas given here are also based entirely upon the function and its derivative at a single square, except for one extra adjacent point.

The coefficients $C_k^{(j)}$ and $D_k^{(j)}$ in (5) were calculated exactly for each j and k by a procedure for complex osculatory interpolation described in (3). The coefficients $A_k^{(j)}$ and $B_k^{(j)}$ in (6) were calculated exactly for each j and k by setting $h = 1$ and then by solving four systems of nine linear equations with complex coefficients (each system corresponding to a j which was 2, $2 + i$, $1 + 2i$ or $2i$), by taking $f(z)$ to be z^m, $m = 0, 1, \cdots, 8$, at $z = 0$, 1, i and $1 + i$ for both $f(z)$ and $f'(z)$ and also at $z = j$ for $f'(z)$ alone. The coefficients in formulas (5) and (6) were checked completely, for every j, by functional checks employing f to be polynomials of every degree ≤ 7 and ≤ 8, respectively. Below are given the four osculatory extrapolation formulas, followed by the four refining formulas, for $j = 2$, $2 + i$, $1 + 2i$ and $2i$:

Extrapolation Formulas

$$f_2 = (-3 - 46i)f_0 + (72 + 4i)f_1 + (-40 - 12i)f_i + (-28 + 54i)f_{1+i} + h\{(6 - 8i)f'_0 + (-12 + 16i)f'_1 + (-8 + 4i)f'_i + (14 - 2i)f'_{1+i}\}$$

$$f_{2+i} = (-40 + 12i)f_0 + (-28 - 54i)f_1 + (-3 + 46i)f_i + (72 - 4i)f_{1+i} + h\{(-8 - 4i)f'_0 + (14 + 2i)f'_1 + (6 + 8i)f'_i + (-12 - 16i)f'_{1+i}\}$$

$$f_{1+2i} = (-40 - 12i)f_0 + (-3 - 46i)f_1 + (-28 + 54i)f_i + (72 + 4i)f_{1+i} + h\{(-4 - 8i)f'_0 + (8 + 6i)f'_1 + (2 + 14i)f'_i + (-16 - 12i)f'_{1+i}\}$$

$$f_{2i} = (-3 + 46i)f_0 + (-40 + 12i)f_1 + (72 - 4i)f_i + (-28 - 54i)f_{1+i} + h\{(-8 + 6i)f'_0 + (4 - 8i)f'_1 + (16 - 12i)f'_i + (-2 + 14i)f'_{1+i}\}$$

Refining Formulas

$$f_2 = \frac{1}{125}[(-175 - 600i)f_0 + (1580 + 60i)f_1 + (-460 - 220i)f_i + (-820 + 760i)f_{1+i} + h\{(44 - 117i)f'_0 + (-176 + 468i)f'_1 + (-96 + 28i)f'_i + (234 + 88i)f'_{1+i} + (24 - 7i)f'_2\}]$$

$$f_i = \frac{1}{125}[(-460 + 220i)f_0 + (-820 - 760i)f_1 + (-175 + 600i)f_i + (1580 - 60i)f_{1+i} + h\{(-96 - 28i)f'_0 + (234 - 88i)f'_1 + (44 + 117i)f'_i + (-176 - 468i)f'_{1+i} + (24 + 7i)f'_{2+i}\}]$$

$$f_{1+2i} = \frac{1}{125}[(-460 - 220i)f_0 + (-175 - 600i)f_1 + (-820 + 760i)f_i + (1580 + 60i)f_{1+i} + h\{(-28 - 96i)f'_0 + (117 + 44i)f'_1 + (-88 + 234i)f'_i + (-468 - 176i)f'_{1+i} + (7 + 24i)f'_{1+2i}\}]$$

$$f_{2i} = \frac{1}{125}[(-175 + 600i)f_0 + (-460 + 220i)f_1 + (1580 - 60i)f_i + (-820 - 760i)f_{1+i} + h\{(-117 + 44i)f'_0 + (28 - 96i)f'_1 + (468 - 176i)f'_i + (88 + 234i)f'_{1+i} + (-7 + 24i)f'_{2i}\}]$$

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Safe Tongs for Laboratory and Shop.—A junior edition of the mechanical manipulator used around atomic reactors and laboratories has been announced by the Atomic Center, New York City. Called "Niptongs," it is designed to grasp and release small radioactive or otherwise dangerous objects at a safe distance from the hands.

Niptongs are used for handling any hot, toxic, or radioactive materials of small dimensions. For instance, they are employed in metal shops for handling small parts being heat-treated, and in chemical laboratories for safe carrying of test tubes and beakers of corrosive acids and for removing crucibles from baking ovens, as well as in radioisotope laboratories in connection with radioactive materials. Additional uses are in food handling processes, in assembling operations for retrieving of small parts, and in many other applications where prevention of bodily contact, or extension of reach is desired. Niptongs are available in 12, 24, 36, and 48 in. lengths.

These simple but sturdy tongs consist of an insulated handle, stationary arm, and spring-loaded, movable arm. Right angle bends at the ends of the two arms act as fingers. They are normally held firmly closed by spring pressure and are easily separated by squeeze-to-release hand action. The entire operation is performed with one hand, leaving the other free for handling covers, doors, other equipment, etc.

Ultrasonic Transducer.—The General Ultrasonics Company of Hartford, Conn., announces a new fully-protected ultrasonic transducer for cleaning, degreasing, pickling, plating and other physical and chemical processes. The type T-2 Sonicell * is completely sealed, metal jacketed, stainless steel lined and thermostatically protected. Its stainless steel treatment chamber permits use of a wide variety of process liquids. Provision is made for draining and recirculating process liquids during operation.

The type T-2 Sonicell * treatment chamber is 20 in. long, 8 in. wide and 6 in. deep with a liquid capacity of 2½ gal. Its resonant frequency of 20 Kc./sec. provides maximum cavitation energy above the range of human audible perception. It is powered by this company's Model 400 Ultrasonic Generator which is continually variable in frequency from 10 Kc. to 1200 Kc. and which has an output of 400 watts.

Magnetic Bobbin Cores.—A new line of tape-wound magnetic bobbin cores, with high standard uniformity, has been announced by the Electronic Instruments Division, Burroughs Corporation, Philadelphia. Known as BIMAGS, the cores are intended for use in switching circuits, shift registers, coincident current matrix systems, pulse transformers, and static magnetic memory elements. The high standard of uniformity achieved in BIMAGS is designed to simplify the task of the design engineer in applying magnetic cores as working components in electronic systems.

BIMAGS are the result of a long range program undertaken by Burroughs which has included a complete study of desired performance ranges, and a re-examination of magnetic materials and manufacturing techniques.

A wide range of standard ¼-in. and ⅛-in. cores are now available in Moly Permalloy (thicknesses of 0.00025 in. and 0.0005 in.) and Orthonik (thicknesses of 0.00025 in. and 0.0005 in.). Other cores of various widths, thicknesses, and bobbin sizes or materials are available for special application.

* Trademark—The General Ultrasonics Company.

THE RELATIVISTIC LINEAR HARMONIC OSCILLATOR

BY

ROBERT PENFIELD [1] AND HENRY ZATZKIS [2]

In the early years of the Bohr atomic theory, A. Sommerfeld gave a derivation of the Kepler motion of the orbital electrons within an atom, taking into account the relativistic variability of mass, and thus arriving at his determination of the fine structure constant. It is the purpose of this paper to investigate the effects of mass variability on the motion of a mass in the field of an elastic force. Some characteristic differences from the behavior of the classical oscillator will appear.

We shall use the following notation:

x = displacement from equilibrium position
a = amplitude of vibration
v_0 = maximal velocity as vibrating mass passes through the equilibrium position
s = elastic force constant
m_0 = mass of particle, when at rest at equilibrium point
v = velocity of particle
ω_0 = classical angular frequency $= \sqrt{\frac{s}{m_0}}$
t = time
c = velocity of light
T = period of vibration

All other symbols will be explained as they are introduced in the context of the paper.

The most convenient approach is the use of the energy principle which holds true in relativistic dynamics also. The decrease in kinetic energy L is equal to the increase in potential energy V, where

$$L = m_0c^2\left\{\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right\}, \tag{1}$$

and

$$V = \tfrac{1}{2}sx^2. \tag{2}$$

Applying the law of conservation of energy, we obtain (the changes

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in energy being measured from the equilibrium position as reference point)

$$\Delta V = -\Delta L$$

or

$$\tfrac{1}{2}sx^2 = m_0c^2\left\{\frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}} - \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right\}. \tag{3}$$

We introduce here the amplitude a which is the value of x, when $v = 0$, that is,

$$\tfrac{1}{2}sa^2 = m_0c^2\left\{\frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}} - 1\right\} \tag{4}$$

or

$$\frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}} = \frac{sa^2}{2m_0c^2} + 1 = \frac{\omega_0^2a^2 + 2c^2}{2c^2}. \tag{5}$$

From Eqs. 1 and 5 it follows that

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\omega_0^2(a^2 - x^2) + 2c^2}{2c^2}. \tag{6}$$

It is convenient to introduce a dimensionless distance z, defined by $z = \frac{x}{a}$ and a dimensionless time quantity τ, defined by $\tau = \omega_0 t$. In terms of these quantities, Eq. 6 reads, if solved for the "velocity" $dz/d\tau$

$$\frac{dz}{d\tau} = \frac{c\sqrt{(1-z^2)[\omega_0^2a^2(1-z^2) + 4c^2]}}{\omega_0^2a^2(1-z^2) + 2c^2}, \tag{7}$$

and

$$\tau = \frac{1}{c}\int_{w=0}^{w=z} \frac{\omega_0^2a^2(1-w^2) + 2c^2}{\sqrt{(1-w^2)[\omega_0^2a^2(1-w^2) + 4c^2]}}\,dw. \tag{8}$$

Equation 8 shows that the time is an incomplete elliptic integral as a function of the displacement or conversely, that the displacement is an elliptic function of the time. The period of the vibration is given by integrating Eq. 7 between the limits $z = 0$ and $z = 1$:

$$T = \frac{4}{c\omega_0}\int_{z=0}^{z=1} \frac{\omega_0^2a^2(1-z^2) + 2c^2}{\sqrt{(1-z^2)[\omega_0^2a^2(1-z^2) + 4c^2]}}\,dz. \tag{9}$$

This integral can be simplified by putting $z = \sin\theta$:

$$T = \frac{4}{c\omega_0}\int_{\theta=0}^{\theta=\pi/2} \frac{\omega_0^2 a^2 \cos^2\theta + 2c^2}{\sqrt{4c^2 + \omega_0^2 a^2 \cos^2\theta}}\, d\theta. \tag{10}$$

This integral can be reduced to a sum of two elliptic integrals, *viz.*

$$T = \frac{4A}{c\omega_0}\int_0^{\pi/2} \sqrt{1 - k^2\sin^2\theta}\, d\theta - \frac{8c}{A\omega_0}\int_0^{\pi/2} \frac{1}{\sqrt{1 - k^2\sin^2\theta}}$$

$$\times\, d\theta = \frac{4}{Ac\omega_0}\left[A^2E(k) - 2c^2K(k)\right], \tag{11}$$

where

$$k^2 = \frac{\omega_0^2 a^2}{4c^2 + \omega_0^2 a^2},$$

and

$$A^2 = 4c^2 + \omega_0^2 a^2.$$

K is the complete elliptic integral of the first kind and E is the complete elliptic integral of the second kind. Equation 11 is similar in structure to the formula for the period T^* of a pendulum of length L of finite angular amplitude θ_0:

$$T^* = \frac{2T_0}{\pi}K(k'), \tag{12}$$

with $k' = \sin\left(\frac{\theta_0}{2}\right)$ and $T_0 = 2\pi\sqrt{\frac{L}{g}}$ = period of a vibration with very small amplitude.

However, there are significant differences between Eqs. 11 and 12.

In the first place, the period depends on two different elliptic integrals. But more important, in the case of Eq. 12, the period T^* depends only on the amplitude θ_0 through k', whereas in the relativistic case of Eq. 11, k is not only a function of the amplitude a but also of the classical frequency ω_0 itself. And this holds true even in the lowest approximation, when Eqs. 11 and 12 are expanded in a Taylor series and only the first term is retained. We obtain then

$$T = T_0\left(1 + \frac{3\pi^2 a^2}{4c^2 T_0^2}\right), \tag{13}$$

and

$$T^* = T_0\left(1 + \frac{\theta_0^2}{16}\right). \tag{14}$$

One can, purely formally, associate with the harmonic oscillator a pendulum of same frequency ω_0. The angular amplitude is then in the same lowest approximation $\theta_0 = \frac{a}{L}$ and $\frac{\pi^2 a^2}{T_0^2} = \frac{gL\theta_0^2}{4}$. This yields in place of Eqs. 13 and 14 the expressions

$$T = T_0\left(1 + \frac{3gL\theta_0^2}{16c^2}\right), \tag{13a}$$

and

$$T^* = T_0\left(1 + \frac{\theta^2}{16}\right). \tag{14a}$$

If we call $T - T_0 = \Delta T$ and $T^* - T_0 = \Delta T^*$, we see that

$$\frac{\Delta T}{\Delta T^*} = \frac{3gL}{c^2}, \tag{15}$$

which shows that the change in frequency due to mass variability is enormously smaller than the one due to the finiteness of the amplitude.

Finally we can calculate the motion of the particle to the same degree of approximation as Eq. 13. We introduce a parameter λ of smallness, defined by

$$\lambda = \frac{a\omega_0}{2c} \tag{16}$$

hence

$$k^2 = \frac{\lambda^2}{1 + \lambda^2} \tag{17}$$

and if only the first term in the Taylor series is retained, the equality $k^2 = \lambda^2$ holds true.

If Eq. 8 is integrated, using the same approximation in the expansion of the integrand, we obtain

$$\omega_0 t = \theta(1 + \tfrac{3}{4}\lambda^2) + \tfrac{1}{8}\lambda^2 \sin 2\theta. \tag{18}$$

For large values of t, this relation becomes approximately

$$\omega_0 t = \theta(1 + \tfrac{3}{4}\lambda^2), \tag{19}$$

or

$$\theta = \omega_0 t(1 - \tfrac{3}{4}\lambda^2), \tag{20}$$

and

$$\sin\theta = \frac{x}{a} = \sin[\omega_0(1 - \tfrac{3}{4}\lambda^2)t], \tag{21}$$

that is,

$$x = a\sin[\omega_0(1 - \tfrac{3}{4}\lambda^2)t]. \tag{22}$$

This result is in agreement with Eq. 13, as can easily be verified by direct substitution.

Acknowledgment:

The authors want to thank the JOURNAL reviewers for calling their attention to a recent publication which deals with the same problem. The reference is:

Det Rgl Danske Videnskabernes Selskab Festschrift til Niels Bohr PÅ Hans 70—Års Dag 7, Oktober, 1955 (Anniversary volume to Niels Bohr), Article No. 10 by Møller.

"World Record" for Strength of Materials.—A new "world record" for the strength of metals has been established by metallurgists at the General Electric Research Laboratory, Schenectady, N. Y. Recent modifications of GE's methods of making "perfect" iron crystals have produced tiny whiskers with tensile strengths as high as 1,900,000 psi., according to Dr. J. Herbert Hollomon, manager of metallurgy and ceramics research at the GE laboratory.

This strength is more than 150 times that of ordinary iron crystals and four times that of the strongest steel wire, Hollomon said. (Tensile strength, also known as ultimate strength, is a relation between the maximum load the material can support and the cross-sectional area of the test sample.)

Hollomon explained today that although present perfect crystals are too small to be "practical," recent studies have shown that there "is a possibility that we will be able to make large crystals free of defects—and thus equally strong."

"Crystal-ball gazers, including some leading scientists, have based many of their prophecies for the future—for aircraft, rockets, skyscrapers, everything made of metal—on the hope that such superstrong materials could be made," Hollomon said. "The day of the wire-like girder and the indestructible automobile is a long way off, but the recent work of our group shows that it may someday come."

Credit for the most recent achievement in perfect crystals was given by Hollomon to Sidney S. Brenner, who works in the Laboratory's chemical metallurgy section, headed by Dr. David Turnbull.

General Electric scientists first announced two years ago that they had made extremely strong iron whiskers. Subsequently the methods of "growing the whiskers" have been adapted to a variety of other metals, including gold, silver, platinum, nickel, and copper. Copper crystals have been made with tensile strengths of 600,000 psi., 20 times as strong as ordinary annealed copper.

The newest iron crystals, less than an inch long and not as big around as a human hair, have nearly twice the strength of the earlier laboratory-made iron whiskers.

Some other typical tensile strengths for well known materials are:

	Psi.
Cast iron	13,000 to 33,000
Aluminum wire	40,000
Copper wire, hard drawn	70,000
Bronze	75,000
Steel	40,000 to 330,000
Piano wire	390,000
Tungsten wire (maximum)	600,000

"Metals are ordinarily from 10 to 1000 times weaker than they theoretically should be," Hollomon said. "This is due to imperfections in the crystals that constitute their basic structure. In the so-called perfect crystals, imperfections are almost eliminated, and metals achieve strengths approaching those indicated by theory.

"Current theory is that the maximum strength for iron should be in the vicinity of 2 to 3 million psi.; thus the recent achievement of making iron crystals with strengths of 1,900,000 psi. is a long step forward and an extremely significant accomplishment," he added.

MATRIX THEORY OF SKIN EFFECT IN LAMINATIONS

BY

LOUIS A. PIPES [1]

SYNOPSIS

This paper presents a mathematical technique for determining the magnetic and electric field intensities and the current density in plane conducting metal plates of constant permeability produced by an external impressed alternating magnetic field. The fundamental Maxwell field equations are used as the basis of the analysis and expressions for the amplitudes of the fields and currents are written in matrix form. It is demonstrated how the field and current distribution may be simply and systematically determined in composite laminated plates such as those used in transformers by a simple matrix multiplication procedure. The technique simplifies the calculations involving skin effect and magnetic shielding.

I. INTRODUCTION

When the permeability of a plane homogeneous conducting slab of metal is constant, independent of the field strength, the eddy currents induced in the slab by time-varying impressed magnetic and electric fields can be calculated accurately (1–4).[2] The effect of these eddy currents is to shield the material of the slab and reduce the intensity of the electric and magnetic fields in the interior of the slab so that the electric and magnetic fields are more intense near the surface of the slab. This phenomenon is known in the electrical literature as "skin effect."

The computation of the current density and the electric and magnetic fields in the interior of a semi-infinite homogeneous slab produced by a harmonically varying magnetic field at its surface is a relatively simple matter. The computation of the current density and field distribution inside a plane slab of metal having a laminated structure, is however, very tedious (5).

In the present paper, a method for the calculation of the field and current distribution in composite slabs is presented. The method is based on a matrix multiplication procedure and greatly reduces the amount of labor involved in the computations.

II. THE BASIC FIELD EQUATIONS

Consider an infinite slab of material of thickness S and of electric conductivity σ, magnetic inductive capacity μ, and electric inductive capacity K. The magnetic intensity $\bar{H}$, the electric intensity $\bar{E}$, and the current density $\bar{J}$ in the slab satisfy the basic Maxwell equations of

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

electromagnetic field theory. In a rationalized MKS system of units (6), these equations have the following form,

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}, \quad \nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} \tag{1}$$

$$\nabla \cdot \bar{B} = 0, \quad \nabla \cdot \bar{D} = \rho. \tag{2}$$

Since the slab is assumed to be a homogeneous isotropic medium, we have the additional relations,

$$\begin{aligned} \bar{D} &= K\bar{E} \\ \bar{B} &= \mu\bar{H} \\ \bar{J} &= \sigma\bar{E}. \end{aligned} \tag{3}$$

In this system of units we have,

$\bar{E}$ = the electric intensity (volts/meter)
$\bar{B}$ = the magnetic induction (webers/meter)
$\bar{D}$ = the electric displacement (coulombs/meter^2)
$\bar{H}$ = the magnetic intensity (amperes/meter)
J = the current density (amperes/meter^2)
σ = the electric conductivity (1/ohm-meter)
$K = K_r K_0$ = the electric inductive capacity of the slab
$\mu = \mu_r \mu_0$ = the magnetic inductive capacity of the slab
K_r = the dielectric constant of the slab
μ_r = the permeability of the slab
$K_0 = 8.85 \times 10^{-12}$ (farads/meter)
$\mu_0 = 4\pi \times 10^{-7} = 1.257 \times 10^{-6}$ (henrys/meter)
ρ = charge density (coulombs/meter^3).

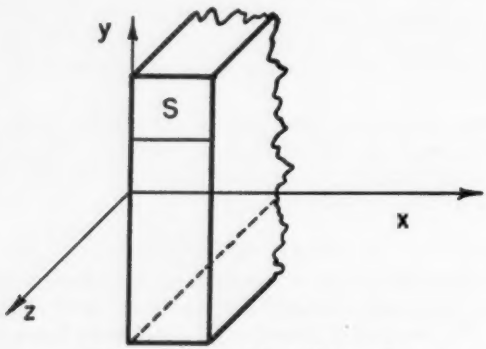

FIG. 1. Infinite slab of thickness S.

Let a magnetic field in the y direction (see Fig. 1), of the form,

$$\bar{H} = \mathrm{j}H_y \tag{4}$$

where **j** is a unit vector in the y direction, be applied to the slab of Fig. 1. Let the magnetic intensity H_y be a periodic quantity in time whose amplitude is a function of the x coordinate alone so that H_y has the following form,

$$H_y = H(x) \cos (wt) = \text{Re } H(x)e^{jwt}. \tag{5}$$

In Eq. 5 $j = (-1)^{\frac{1}{2}}$, e is the base of natural logarithms and Re denotes "the real part of." The second Maxwell equation (1) may be written in the form,

$$\nabla \times \bar{H} = \sigma \bar{E} + \frac{\partial \bar{D}}{\partial t}. \tag{6}$$

It can be shown (7) that if the frequency f of the oscillation of H_y, $f = w/2\pi$ of the impressed magnetic field is such that the wave-length λ given by,

$$\lambda = c/f = 2.998 \times 10^8/f \quad \text{(meters)} \tag{7}$$

is large in comparison with the thickness of the slab, then Maxwell's "displacement current" $\partial \bar{D}/\partial t$ may be neglected. In such a case, Eq. 6 may be solved for the electric intensity $\bar{E}$ in the following form,

$$\bar{E} = (\nabla \times \bar{H})/\sigma = \frac{\mathbf{k}}{\sigma} \frac{\partial H_y}{\partial x}, \tag{8}$$

where **k** is the unit vector in the z direction. The electric intensity therefore has the magnitude,

$$E_z = \frac{1}{\sigma} \frac{\partial H_y}{\partial x}. \tag{9}$$

The first Maxwell equation (1) may be written in the form,

$$\nabla \times \bar{E} = \nabla x(\mathbf{k}E_z) = -\mathbf{j} \frac{\partial E_z}{\partial x} = -\mu \frac{\partial \bar{H}}{\partial t} = -j\mu \frac{\partial H_y}{\partial t}, \tag{10}$$

where μ is the magnetic inductive capacity of the slab. Hence as a consequence of Eq. 10, it follows that,

$$\frac{\partial E_z}{\partial x} = \mu \frac{\partial H_y}{\partial t}. \tag{11}$$

Equations 9 and 11 serve to determine the magnetic and electric fields in the interior of the slab in terms of the fields impressed at its surface. Associated with the electric field E_z in the slab there exists a current density of magnitude,

$$J_z = \sigma E_z = \frac{\partial H_y}{\partial x} \quad \text{(amperes/meter}^2\text{)}. \tag{12}$$

This is the magnitude of the sheet of alternating current in the interior of the slab.

III. THE SKIN EFFECT IN A SINGLE SLAB: THE TRANSFER MATRIX

In order to determine the electric and magnetic fields and the current density in the interior of the slab of Fig. 1 produced by an impressed oscillatory magnetic field on the surface $x = 0$ of the slab of the following form,

$$(H_y)_{x=0} = H_1 \cos (wt), \tag{13}$$

let

$$H_y = \text{Re}[H(x)e^{jwt}] \tag{14}$$

and

$$E_z = \text{Re}[E(x)e^{jwt}]. \tag{15}$$

If Eqs. 14 and 15 are substituted into Eqs. 9 and 11, the resulting equations may be written in the following form,

$$E(x) = \frac{1}{\sigma}\frac{dH}{dx} \tag{16}$$

$$\frac{dE}{dx} = jw\mu H. \tag{17}$$

$E(x)$ may be eliminated between (16) and (17) and the following equation obtained,

$$\frac{d^2H}{dx^2} = jw\mu\sigma H. \tag{18}$$

The general integral of this equation is

$$H(x) = A \cosh (ax) + B \sinh (ax), \tag{19}$$

where A and B are arbitrary constants and the quantity a is given by,

$$a = (jw\mu\sigma)^{\frac{1}{2}} = (1 + j)k, \tag{20}$$

where

$$k = (w\mu\sigma/2)^{\frac{1}{2}}. \tag{21}$$

As a consequence of (16), $E(x)$ may be written in the form,

$$E(x) = \frac{1}{\sigma}\frac{dH}{dx} = \frac{a}{\sigma}[A \sinh (ax) + B \cosh (ax)]. \tag{22}$$

Let the following boundary conditions be assumed for $H(x)$ and $E(x)$,

$$H(0) = H_1, \quad E(0) = E_1. \tag{23}$$

In terms of these boundary conditions, the following values of A and B are determined,

$$A = H_1, \quad B = \sigma \frac{E_1}{a}. \tag{24}$$

The expressions for $H(x)$ and $E(x)$ given by (19) and (22) may be written in the convenient matrix form (8),

$$\begin{bmatrix} H(x) \\ E(x) \end{bmatrix} = \begin{bmatrix} \cosh(ax) & m \sinh(ax) \\ \dfrac{\sinh(ax)}{m} & \cosh(ax) \end{bmatrix} \cdot \begin{bmatrix} H_1 \\ E_1 \end{bmatrix}, \tag{25}$$

where

$$m = \frac{\sigma}{a} = (\sigma/jw\mu)^{\frac{1}{2}}. \tag{26}$$

If $x = S$, Eq. 25 gives the following values for $H(S) = H_2$ and $E(S) = E_2$,

$$\begin{bmatrix} H_2 \\ E_2 \end{bmatrix} = \begin{bmatrix} \cosh(\theta) & m \sinh(\theta) \\ \dfrac{\sinh(\theta)}{m} & \cosh(\theta) \end{bmatrix} \cdot \begin{bmatrix} H_1 \\ E_1 \end{bmatrix}, \tag{27}$$

where,

$$\theta = aS = S(jw\mu\sigma)^{\frac{1}{2}}. \tag{28}$$

H_2 and E_2 are the magnitudes of the magnetic and electric intensities at the surface $x = S$ of the slab.

The matrix,

$$[T] = \begin{bmatrix} \cosh(\theta) & m \sinh(\theta) \\ \dfrac{\sinh(\theta)}{m} & \cosh(\theta) \end{bmatrix} \tag{29}$$

will be called the "transfer matrix" of the slab. This matrix has the following inverse,

$$[T]^{-1} = \begin{bmatrix} \cosh(\theta) & -m \sinh(\theta) \\ \dfrac{-\sinh(\theta)}{m} & \cosh(\theta) \end{bmatrix}. \tag{30}$$

Equations 25, 27 and 30 are very useful for the computations of the electric and magnetic fields inside a single slab when the field distribution at its surface is given. It will be shown in a later section that by means of matrix multiplication of transfer matrices the field distribution in

composite slabs of laminated type may be obtained with an economy of effort.

IV. FIELD DISTRIBUTION INSIDE A VERY THICK SLAB

The well-known result for the field distribution inside a very thick slab will be obtained by specializing the results of Section III. In a very thick slab S is large and it can be assumed that the quantity θ defined by

$$\theta = aS = S(1 + j)(w\mu\sigma/2)^{\frac{1}{2}} \tag{31}$$

has such a large real part that the following approximations can be made,

$$\sinh(\theta) \doteq \frac{e^{\theta}}{2}, \quad \cosh(\theta) \doteq \frac{e^{\theta}}{2}. \tag{32}$$

If these approximations are used in (27) and the ratio of H_1 to E_1 is computed, the following result is obtained,

$$\frac{H_1}{E_1} = -m. \tag{33}$$

If $E_1 = -H_1/m$ is substituted into (25) and the matrix equation is expanded, the result is,

$$H(x) = H_1 e^{-ax}, \tag{34}$$

and

$$E(x) = -H_1 \frac{e^{-ax}}{m} = -(jw\mu/\sigma)^{\frac{1}{2}} H_1 e^{-ax}. \tag{35}$$

The magnetic field in the interior of the slab produced by the impressed magnetic field $H_y = H_1 \cos(wt)$ on the surface $x = 0$ of the slab, is given by Eq. 14 in the form,

$$H_y = \mathrm{Re}[H_1 e^{-kx(1+j)} e^{jwt}] = H_1 e^{-kx} \cos(wt - kx). \tag{36}$$

The induced electric field is given by Eq. 15 in the form,

$$E_z = \mathrm{Re}[E(x) e^{jwt}] = -(w\mu/\sigma)^{\frac{1}{2}} H_1 e^{-kx} \cos(wt - kx + \pi/4). \tag{37}$$

The current density J_z is given by the equation,

$$J_z = \sigma E_z = -(w\mu\sigma)^{\frac{1}{2}} H_1 e^{-kx} \cos(wt - kx + \pi/4). \tag{38}$$

These are the well-known results for the induced fields and current densities in a very thick slab produced by an impressed alternating magnetic field at its surface.

V. FIELD DISTRIBUTION IN A PLATE OF FINITE THICKNESS

Consider a plate of finite thickness S as shown in Fig. 1, and let an alternating magnetic field in the y direction be impressed at the surface of the plate so that,

$$(H_y)_{x=0} = H_0 \cos(wt) \quad \text{and} \quad (H_y)_{x=0} = H_0 \cos(wt). \tag{39}$$

It is desired to determine the magnitudes of the magnetic and electric fields inside the plate and to determine the current density induced inside the plate. To facilitate the algebra, write the transfer matrix (29) in the following form,

$$[T] = \begin{bmatrix} \cosh(\theta) & m \sinh(\theta) \\ \dfrac{\sinh(\theta)}{m} & \cosh(\theta) \end{bmatrix} = \begin{bmatrix} A & B \\ C & A \end{bmatrix}. \tag{40}$$

In terms of the elements A, B, C of the transfer matrix, the complex amplitudes E_1 and E_2 of the electric intensities at the surfaces of the plate may be written in the form,

$$\begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} -A/B & 1/B \\ -1/B & A/B \end{bmatrix} \cdot \begin{bmatrix} H_1 \\ H_2 \end{bmatrix}. \tag{41}$$

This follows from (27). In this case, since $H_1 = H_2 = H_0$ (41) gives the following result,

$$E_1 = -\frac{H_0}{m} \tanh(\theta/2). \tag{42}$$

In terms of H_0, the complex amplitudes of the magnetic and electric fields in the interior of the plate are given by (25) in the form,

$$\begin{aligned} H(x) &= H_0 [\cosh(ax) - \tanh(\theta/2) \sinh(ax)] \\ E(x) &= \frac{H_0}{m} [\sinh(ax) - \tanh(\theta/2) \cosh(ax)]. \end{aligned} \tag{43}$$

The complex amplitude of the magnetic field at the center of the plate is determined from (43) in the form,

$$H_m = H(S/2) = H_0 [\cosh(\theta/2) - \tanh(\theta/2) \sinh(\theta/2)]. \tag{44}$$

After some reductions, this expression may be writtin in the form,

$$H_m = H_0/\cosh(\theta/2). \tag{45}$$

Since, $\theta = kS(1 + j)$, we have,

$$\cosh(\theta/2) = \cosh(kS/2)\cos(kS/2) + j\sinh(kS/2)\sin(kS/2) \quad (46)$$

and

$$|\cosh(\theta/2)| = \frac{[\cosh(kS) + \cos(kS)]^{\frac{1}{2}}}{2^{\frac{1}{2}}}. \quad (47)$$

Hence if (47) is substituted into (45) the result is,

$$H_m = \sqrt{2}H_0 e^{-j\phi}/[\cosh(kS) + \cos(kS)]^{\frac{1}{2}}, \quad (48)$$
$$\tan(\phi) = \tanh(kS/2)\tan(kS/2).$$

Therefore the instantaneous value of the magnetic field at the center of the plate is

$$(H_y)_{x=S/2} = \sqrt{2}H_0\cos(wt - \phi)/[\cosh(kS) + \cos(kS)]^{\frac{1}{2}} \quad (49)$$
$$k = (w\mu\sigma/2)^{\frac{1}{2}}.$$

The complex amplitude of the current density in the interior of the plate is given by the equation,

$$J(x) = \frac{\sigma H_0}{m}[\sin(ax) - \tanh(\theta/2)\cosh(ax)]. \quad (50)$$

VI. THE TRANSFER MATRIX OF AN INSULATING SLAB

Consider an infinite plane slab of thickness S and magnetic inductive capacity μ but of zero conductivity. The Maxwell equations (1) for the field distribution in such a slab reduce to

$$\nabla \times \bar{E} = -\mu\frac{\partial \bar{H}}{\partial t}, \quad \nabla \times \bar{H} = 0. \quad (51)$$

In terms of the components E_z and H_y, these equations reduce to

$$\frac{\partial E_z}{\partial x} = \mu\frac{\partial H_y}{\partial t}, \quad \frac{\partial H_y}{\partial x} = 0. \quad (52)$$

In terms of the complex amplitudes $E(x)$ and $H(x)$, Eqs. 52 give the following relations,

$$\frac{dE}{dx} = jw\mu H, \quad \frac{dH}{dx} = 0. \quad (53)$$

If the boundary conditions, $E(0) = E_1$, $E(S) = E_2$, $H(0) = H_1$, $H(S) = H_2$ are introduced, the integrals of (53) give the following results,

$$H_2 = H_1, \quad E_2 = jw\mu SH + E_1. \quad (54)$$

These equations may be written in the following matrix form,

$$\begin{bmatrix} H_2 \\ E_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ jw\mu S & 1 \end{bmatrix} \cdot \begin{bmatrix} H_1 \\ E_1 \end{bmatrix}. \tag{55}$$

The square matrix,

$$[T]_0 = \begin{bmatrix} 1 & 0 \\ jw\mu S & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ N & 1 \end{bmatrix}, \quad N = jw\mu S \tag{56}$$

is the transfer matrix of the insulating slab. The transfer matrix of an insulating slab may also be determined by allowing $\sigma \to 0$ in the general transfer matrix (29) by a limiting process. To do this, it may be noticed that,

$$\operatorname*{Limit}_{\sigma \to 0} \theta = 0, \quad \operatorname*{Limit}_{\sigma \to 0} \frac{a}{\sigma} \sinh(\theta) = jw\mu S, \quad \operatorname*{Limit}_{\sigma \to 0} \frac{\sigma}{a} \sinh(\theta) = 0. \tag{57}$$

The transfer matrix of an insulating slab is useful in calculating the fields and current densities in laminated composite plates composed of layers of insulating and conducting materials as used in the cores of transformers.

VII. THE SKIN EFFECT IN COMPOSITE LAMINATED SLABS

The full power of the matrix formulation of the equations that govern the distribution of the fields and currents inside blocks of metal becomes apparent if it is desired to compute the field and current distribution in a composite block composed of n parallel slabs or laminations of different electrical characteristics as shown in Fig. 2. If μ_k

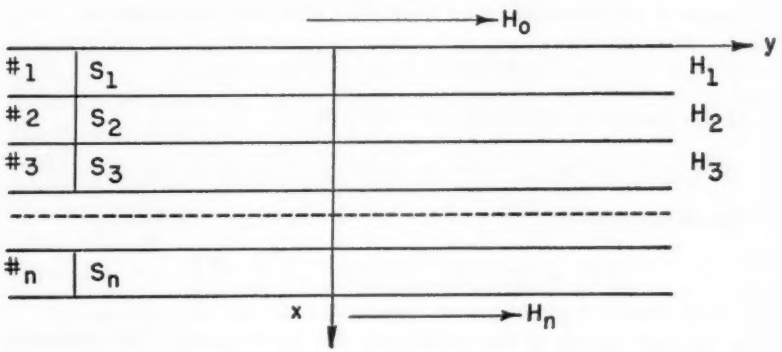

FIG. 2. Composite slab.

and σ_k denote the magnetic inductive capacity and the conductivity of the k^{th} lamination and S_k its thickness, then the square matrix,

$$[T_k] = \begin{bmatrix} \cosh(\theta_k) & m_k \sinh(\theta_k) \\ \dfrac{\sinh(\theta_k)}{m_k} & \cosh(\theta_k) \end{bmatrix} \tag{58}$$

where,

$$\theta_k = S_k(jw\mu_k\sigma_k)^{\frac{1}{2}}, \quad m_k = (\sigma_k/jw\mu_k)^{\frac{1}{2}} \tag{59}$$

is the transfer matrix of the k^{th} lamination. Let a column matrix (F_k) be defined by the equation

$$(F_k) = \begin{bmatrix} H_k \\ E_k \end{bmatrix}, \tag{60}$$

where H_k is the complex amplitude of the magnetic field and E_k is the complex amplitude of the electric field at the interface between the k^{th} and the $(k+1)^{th}$ laminations, we have as a consequence of (27),

$$(F_{k+1}) = [T_{k+1}](F_k), \quad k = 0, 1, 2, 3, \cdots (n-1). \tag{61}$$

It can be shown from general electromagnetic theory (4) that the tangential components of the electric field $\bar{E}$ and of the magnetic field $\bar{H}$ are continuous at the interfaces between laminations and hence (61) may be used to obtain the complex magnitudes of the electric and magnetic fields E_k and H_k by a process of matrix multiplication. For example, we have as a consequence of (61),

$$(F_1) = [T_1](F_0) \tag{62}$$

$$(F_2) = [T_2](F_1). \tag{63}$$

Hence if (F_1) is substituted from (62) into (63), the result is

$$(F_2) = [T_2][T_1](F_0). \tag{64}$$

If the process is continued, the result is

$$(F_n) = [T_n][T_{n-1}][T_{n-2}]\cdots[T_1](F_0). \tag{65}$$

The square matrix,

$$[T] = [T_n][T_{n-1}]\cdots[T_1] = \begin{bmatrix} A & B \\ B & C \end{bmatrix} \tag{66}$$

is the transfer matrix of the entire slab of n laminations. The external fields of the slab H_0, E_0 and H_n and E_n are related by the equations,

$$\begin{bmatrix} H_n \\ E_n \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} H_0 \\ E_0 \end{bmatrix}, \quad \text{or} \quad \begin{bmatrix} H_0 \\ E_0 \end{bmatrix} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \cdot \begin{bmatrix} H_n \\ E_n \end{bmatrix}. \tag{67}$$

If the complex amplitudes of the magnetic field H_0 and H_n at the surfaces of the composite slab are given, then the amplitudes E_0 and E_n of the electric fields at the surfaces of the slab are given by

$$\begin{bmatrix} E_0 \\ E_n \end{bmatrix} = \frac{1}{B} \begin{bmatrix} -A & 1 \\ -1 & D \end{bmatrix} \cdot \begin{bmatrix} H_0 \\ H_n \end{bmatrix}. \tag{68}$$

Block Composed of Sandwich Laminations

Consider a composite slab of a "sandwich type" as shown in Fig. 3.

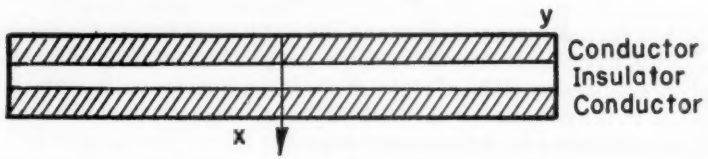

FIG. 3. Sandwich slab.

The transfer matrix of this slab is,

$$[T] = \begin{bmatrix} \cosh(\theta) & m \sinh(\theta) \\ \dfrac{\sinh(\theta)}{m} & \cosh(\theta) \end{bmatrix} \cdot \begin{bmatrix} 1 & \\ jw\mu S_i & 1 \end{bmatrix}^n \cdot \begin{bmatrix} \cosh(\theta) & m \sinh(\theta) \\ \dfrac{\sinh(\theta)}{m} & \cosh(\theta) \end{bmatrix},$$

$$= \begin{bmatrix} A & B \\ C & A \end{bmatrix} \tag{69}$$

where,

$$\theta = S_c(jw\mu\sigma)^{\frac{1}{2}}, \quad m = (\sigma/jw\mu)^{\frac{1}{2}}. \tag{70}$$

S_c and S_i denote the thicknesses of the conducting and insulating slabs, respectively. The over-all transfer matrix $[T_0]$ of a plane block composed of n insulating slabs and n conducting slabs of thicknesses $2S_c - S_i - 2S_c - S_i - \cdots 2S_c$ is,

$$[T_0] = \begin{bmatrix} \cosh(\theta) & m \sinh(\theta) \\ \dfrac{\sinh(\theta)}{m} & \cosh(\theta) \end{bmatrix} \cdot \begin{bmatrix} A & B \\ C & A \end{bmatrix}^n \cdot \begin{bmatrix} \cosh(\theta) & m \sinh(\theta) \\ \dfrac{\sinh(\theta)}{m} & \cosh(\theta) \end{bmatrix}. \tag{71}$$

In order to raise the middle matrix of the chain of matrices in (71) to the power n, let

$$a = \cosh^{-1}(A) \tag{72}$$

and

$$Z_0 = (B/C)^{\frac{1}{2}}; \tag{73}$$

then,

$$\begin{bmatrix} A & B \\ C & A \end{bmatrix}^n = \begin{bmatrix} \cosh(an) & Z_0 \sinh(an) \\ \dfrac{\sinh(an)}{Z_0} & \cosh(an) \end{bmatrix}. \tag{74}$$

This matrix result is well-known. A simple proof of it is found in (9). Once the over-all transfer matrix of the entire composite block is obtained, the field distribution inside the block produced by impressed alternating magnetic fields at its surfaces may be calculated by a sequence similar to that of (61).

VIII. CONCLUSION

It has been demonstrated in the above discussion that the formulation of the basic equations that govern the phenomenon of "skin effect" in plane conducting slabs of metal in matrix notation, reduces the labor involved in the calculation of the electric and magnetic field distribution in these structures by a significant degree.

The power of the matrix formulation is particularly evident in the calculation of the field and current distribution in laminated blocks. By the use of matrix algebra, the computations are reduced to matrix multiplication and the labor of evaluating the arbitrary constants at the interfaces is avoided.

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TORQUE ANALYSIS OF THE SCHRAGE COMMUTATOR MOTOR

BY

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ABSTRACT

A relation between the torque and the primary leakage reactance of the Schrage commutator motor is pointed out and it has been shown that an increase in this primary reactance results in an extension of the supersynchronous zone with no significant effect on the subsynchronous region.

Assuming a linear relation between the slip and the brush separation the expression for the torque has been maximized. Quadratic equations in the variable "slip" have been obtained and the physical significance of the optimum roots has been discussed in detail.

INTRODUCTION

The torque analysis of the Schrage motor (**5**)[2] as developed in this paper will perhaps merit a short description of the motor in the beginning in order to introduce the results of the analysis. The Schrage motor is a commutator induction motor which combines speed control and power factor improvement facilities. The commutator with the lap winding is in parallel with an induction motor and this has necessitated the machine to be rotor-fed. This special feature of the machine has given it a shunt characteristic. The basic design features of it are as follows: The primary, an usual three-phase winding, is built on the rotor core into which is also housed the commutator and the drum type lap winding. As a rule, this compensating winding is on the top of the primary in the same rotor slots and thus they have practically no leakage among themselves. For a three-phase winding the number of brushes on the commutator will be six, as each brush is broken up into halves. One set of three halves will be put on one brush rocker and the other set of three halves on a separate rocker. The secondary winding is placed in the stator in contra-distinction to the usual practice and the three phases of the stator are kept entirely separate one from another and six leads are brought out for connection to the six half-brushes.

In the case of an ordinary induction motor the rotor runs in the same direction as the field due to the stator winding, but, here the rotor which carries the primary runs against the primary field, so that at synchronous speed there is no relative motion of the primary field and the secondary conductors. Since the air gap field is produced from the primary field and the motion of the primary rotor conductors, this field can stand

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

still in space or rotate forwards and backwards depending upon the direction of rotation of the rotor and its speed of rotation. Leaving aside the case of reverse speed running, normally there will be two speed zones, namely, the subsynchronous and supersynchronous with a synchronous speed at the boundary. This would also be apparent from the fact that energy transference can occur from and to the secondary via the brushes because the voltages at the half-brushes and at the stator terminals are at the same slip frequency though, of course, the voltage generated in the lap winding will be of the supply frequency. In the case of energy being transferred from the secondary the speeds will be subsynchronous and when energy will be transferred to the secondary the speeds will be supersynchronous. In both these speed ranges the e.m.f.'s across the brush halves and the stator terminals will be of identical polarity and hence will oppose[3] each other. This explains why it is necessary to cross over the half-brushes in the supersynchronous zone. At subsynchronous speeds the secondary e.m.f. exceeds the e.m.f. between pairs of half-brushes in order that the lap-primary combination may accept energy from the stator and at supersynchronous speeds the reverse thing happens, that is, the brush e.m.f. now exceeds the secondary e.m.f. The half-brushes are short-circuited at a speed little below synchronous when the machine runs as an ordinary induction motor. At the synchronous speed the secondary e.m.f. will be zero but the brush e.m.f. will be finite and will deliver d-c. current to the stator. The power factor improvement of the machine is automatic with the increase of speeds because of the fact that the brush e.m.f. leads on the secondary e.m.f. due to leakage reactance between primary or lap and secondary windings but, however, an artificial improvement is still possible by displacing the center line of the brush gear from the common center line of the primary and the secondary.

With this background, it has been possible to show that the torque of a Schrage motor is related to its primary leakage reactance and this torque can also be maximized with respect to the slip of the machine. A short account of the method and results obtained is introduced below before the actual derivations are shown.

The torque of a Schrage motor is found to be related to the primary leakage reactance and with all other variables assumed constant it increases with the increase of this leakage reactance at subsynchronous and supersynchronous speeds. Since the primary leakage reactance has influence over the secondary induced voltage, then the effect of an increase of the primary leakage reactance would be larger torques for the supersynchronous zone, or, in other words an extension of this zone with no significant effect on the subsynchronous zone.

[3] There is a case (3) when these two e.m.f.'s can, instead, add each other instantaneously. This occurs in a narrow zone when the brushes have just crossed over, but the speed of the machine is still subsynchronous.

The expression of torque has been maximized when slip is taken proportional to the amount of brush separation for both subsynchronous and supersynchronous speeds. A relationship of the type $S = ma + C$ has been assumed with S and a representing slip and brush separation and m and C constants. Such a relationship expresses fully the behavior of the machine in both the speed ranges and with this the torque expression can be reduced to the sum of two slip-dependent terms. In the above relationship S and a will be both negative in the supersynchronous zone and in magnitude S is then equal to $ma - C$.

The condition for maximum for the slip-dependent terms of the torque expression leads to quadratic equations in the variable "slip." When m approaches unity and C approaches zero, that is, when $S = a$ (a condition which may be satisfied for practical purposes) then, one particular term of the torque expression gives almost exactly the same value for the maximum condition as that required by an ordinary induction motor. Also, this maximum condition is found to occur in both the subsynchronous and supersynchronous speed ranges and is the same in either. The value of the maximum torque corresponding to the above case is observed to be dependent on secondary resistance and leakage reactance. The other term of the torque expression gives two different values of slip both being negative. This means that either these maximum torques will be in the supersynchronous zone only or in the reverse speed zone because reverse speeds also may be interpreted to correspond to negative slip. However, the corresponding torque values (for $S = a$) show that only one of these is positive and hence is in the supersynchronous zone. This maximum torque is inverse to the secondary resistance and independent of any other thing. The negative maximum torque corresponding to the reverse speed zone is found to be dependent on both secondary resistance and leakage reactance.

TORQUE EQUATION

Torque expression has been determined in terms of the usual machine parameters and the method used is similar to that taken in case of ordinary induction motors. The general equation for the torque may be written as (4, 6)

$$T \propto E_1 \cdot \frac{\overline{aE_L} \sim \overline{SE_1}}{\sqrt{\{(aR_L + R_S)^2 + (SX_2)^2\}}} \cdot \frac{(aR_L + R_S)}{\sqrt{\{(aR_L + R_S)^2 + (SX_2)^2\}}} \qquad (1)$$

with T representing the torque, E_1, the primary induced voltage from the stator, S the slip, R_L and R_S the resistance of the lap and secondary windings, X_2 the leakage reactance of the secondary, E_L the voltage induced in the lap winding between the half-brushes when they are 180 electrical degrees apart and a the ratio of effective lap turns for any brush displacement to that for 180 electrical degrees. It is assumed here that the lap winding has no leakage reactance and the ratio between

the primary and the secondary is unity. The equation shows that the torque is proportional to the air-gap flux which is represented by the voltage E_1, to the secondary current given by the middle expression and to the power factor between the two which is shown by the end expression. The numerator of the current expression has been typically shown due to the fact that the voltage responsible for driving current through the secondary winding is the vector difference of the voltages aE_L and SE_1. Thus, it may be clear that this voltage difference being dependent on the angle (1, 2) between aE_L and SE_1 or in other words on the primary leakage reactance X_1 the torque of the motor with all other things assumed as constant is related to X_1. With increasing X_1 the torque tends to increase. But, since an increase in X_1 will result in the reduction of E_1 when all other things remain unchanged the net voltage difference due to the effect of E_1 in the secondary side for any slip S is reduced in the subsynchronous zone. Hence, due to an increase of X_1 there is definite augmentation of the voltage difference $(\overline{aE_L} - \overline{SE_1})$ in the supersynchronous zone, whereas, in the subsynchronous zone the voltage difference $(\overline{SE_1} - \overline{aE_L})$ due to two opposing factors tends to remain constant. Another aspect of the situation is that with increased values of X_1 a certain speed in the supersynchronous zone can be attained with a smaller brush voltage and this means that with the same brush gear a larger speed range may be possible.

MAXIMUM TORQUE

To find the maximum torque of the motor it is first necessary to maximize the torque with respect to a variable or the variables for which the maximum may exist.

In this connection it may be seen that there are two variables in the torque equation, namely, the brush separation term a and the slip S. These two may be amalgamated if a relationship of the type $S = ma + C$, with m and C as constants is assumed. Let this relation be explained further here in order to justify its suitability. It may be seen that when $a = 0$, $S = C$, which means that at the short circuit position of the brushes S is finite, small and positive. Similarly, when $S = 0$, then, $a = -\frac{C}{m}$, which again means that at zero slip the brush separation is small but negative, that is, the brushes are crossed over. In the supersynchronous zone both S and a will be negative and then S in magnitude is equal to $ma - C$. Thus, the behavior of the motor can be well explained by the above relationship.

Now, with this relationship the secondary voltage term may be approximately reduced as below:

$$\overline{SE_1} - \overline{aE_L} = SE_1\left(1 - \frac{K}{m}\cos\theta\right) + \frac{KCE_1}{m}\cos\theta \qquad (2)$$

in the subsynchronous zone and,

$$\overline{aE_L} - \overline{SE_1} = SE_1\left(\frac{K}{m} - \cos\theta\right) + \frac{KCE_1}{m} \tag{3}$$

in the supersynchronous zone where θ is the angle between E_L and E_1 and K an assumed factor between these two voltages. Hence, the torque may be shown to be dependent on two slip consisting terms, namely,

$$1.\quad \frac{S(aR_L + R_S)}{(aR_L + R_S)^3 + (SX_2)^2}$$

and

$$2.\quad \frac{(aR_L + R_S)}{(aR_L + R_S)^2 + (SX_2)^2}.$$

With $R_L = nR_S$ (n is the ratio of effective lap turns for 180 electrical degrees brush displacement to effective stator turns), if the relations $S = ma + C$ and $S = ma - C$ at the subsynchronous and supersynchronous speeds, respectively, are substituted in the first term above, then the following quadratic equations are obtained after maximization and simplification.

$$0 = S^2(m^2R_S^3n^2 + m^3R_SnCX_2^2 - m^4R_SX_2^2) + S(2m^3R_S^3n) + m^4R_S^3 \tag{4}$$

in the subsynchronous case and,

$$0 = S^2(m^2R_S^3n^2 - 3m^3R_SnCX_2^2 - m^4R_SX_2^2) + S(2m^3R_S^3n) + m^4R_S^3 \tag{5}$$

in the supersynchronous case (products of higher powers of R and C being neglected).

The values of S in these equations may easily be worked out now. But, for simplicity if $S = a$ is put down, that is, if m is taken as unity and C as zero, then from both these equations

$$S = \frac{-R_S^2n \pm R_SX_2}{R_S^2n^2 - X_2^2}. \tag{6}$$

If R_S^2n and $R_S^2n^2$ are now neglected, then S may be written as

$$S = \frac{\pm R_S}{-X_2}. \tag{7}$$

This shows that $S = \dfrac{R_S}{X_2}$ in magnitude and can be both positive and negative which means that this maximum torque occurs at both sub-

synchronous and supersynchronous speed zones at the same value for S. It may be noted here that this maximum condition is exactly the same as that required for an ordinary induction motor. However, for this value of S the maximum torque is found to be proportional to

$$\frac{(nR_S + X_2)}{(nR_S + X_2)^2 + X_2^2}.$$

The other term in the torque equation, namely,

$$\frac{(aR_L + R_S)}{(aR_L + R_S)^2 + (SX_2)^2}$$

was next maximized. Substituting as before the relation of S and a and writing $R_L = nR_S$ this term will reduce to

$$\frac{R_S(nS \mp nC + m)m}{R_S^2[n^2(S \mp C)^2 + m^2 + 2mn(S \mp C)] + m^2S^2X_2^2} \tag{8}$$

the minus sign indicating subsynchronous and the plus sign supersynchronous positions respectively. It may be easily found that d^2T/dS^2 is negative with respect to the above term as it was in the previous case and thus it shows the term has a maximum with respect to S. Putting $dT/dS = 0$ the condition for maximum is given by a quadratic equation as below:

$$\begin{aligned} 0 = S^2(R_S^2n^3 + m^2nX_2^2) + S(2m^3X_2^2 + 2R_S^2mn^2 \\ \mp 2R_S^2n^3C \mp 2Cnm^2X_2^2) + (R_S^2n^3C^2 \\ + R_S^2m^2n \mp 2R_S^2mn^2C). \end{aligned} \tag{9}$$

In this equation if the terms containing the product of R_S^2C or $R_S^2C^2$, etc., are neglected as before, then the equation will reduce to

$$\begin{aligned} 0 = S^2(R_S^2n^3 + m^2nX_2^2) + S(2m^3X_2^2 + 2R_S^2mn^2 \\ \mp 2Cnm^2X_2^2) + R_S^2m^2n. \end{aligned} \tag{10}$$

Ultimately, the solution for this equation at both the speed ranges may be found for the following values of S:

$$S = -\frac{R_S^2n^2m}{R_S^2n^3 + m^2nX_2^2}$$

or,

$$S = -\frac{2m^3X_2^2 + R_S^2mn^2 \mp 2Cnm^2X_2^2}{R_S^2n^3 + m^2nX_2^2}, \tag{11}$$

the term $4m^4R_S^2n^2X_2^2$ being neglected for simplification in a sum of series of terms where the other terms are very much higher. This shows that two maximum values of the torque are possible both lying for negative values of the slip.

Substituting these values for S in the expression $(aR_L + R_S)/(aR_L + R_S)^2 + (SX_2)^2$ it may be seen that for the first value of S the maximum torque will be positive and hence will lie in the supersynchronous zone. But, for the other value of S the torque may even become negative when it obviously would lie in the reverse speed range of the machine. This latter condition will be very much dependent on the values of m, n and C. Had the above expression been maximized for m tending towards unity and C toward zero, that is, if $S = a$ would be taken, the first maximum would depend on $\frac{1}{R_S}$ and the latter on $-\frac{n^2R_S}{n^2R_S^2 + 4X_2^2}$.

It is evident that since the first maximum is positive, it is in the supersynchronous zone and the latter, being negative, can correspond only to the reverse speed zone because the machine is treated as a motor here.

DISCUSSION

The relationship between E_L and E_1 in the voltage term as represented in this paper should not be mistaken to be proportional. It is for the sake of mathematical argument only that E_L is assumed to be a multiple of E_1. Even without splitting the secondary voltage term and taking advantage of this relationship between E_L and E_1, it could be presumed that this voltage term would be nearly constant even though it is a function of slip and brush separation. This is evident from the manner in which it is dependent on slip and brush separation in both subsynchronous and supersynchronous speed ranges. As such, it could be taken that the torque is only proportional to the term

$$\frac{(aR_L + R_S)}{(aR_L + R_S)^2 + (SX_2)^2}.$$

Then, substituting the relationship of S and a in this term and proceeding as before, almost similar results could be obtained. An important outcome, however, of the splitting of the secondary voltage term is that the Schrage machine is also, like the ordinary induction motors, identically susceptible to the variation of voltage.

It may be pointed out that the method of maximization as used here in terms of a function of one variable is not only suited because of the fidelity of the assumed relation between S and a but is also much easier

in view of the difficulty of other methods dealing with functions of many variables in this particular case.

All through these investigations symmetrical brush positions have been assumed.

Acknowledgment

The author sincerely thanks Professor D. S. Kothari, Scientific Adviser, Ministry of Defence, India, for his kind permission to publish the paper.

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NOTES FROM THE NATIONAL BUREAU OF STANDARDS

PLASTIC SPRINGS

A practical procedure for mass-producing plastic springs has been developed by the National Bureau of Standards in work sponsored by the Army Ordnance Corps. Springs formed in this way[1] from glass fiber-reinforced resin have desirable mechanical properties for a wide range of applications. Until now plastic springs have been little used because suitable techniques for making springs of the types needed have been lacking.

Polymeric materials, such as plastics and rubbers, have a number of inherent advantages for use in springs for special purposes. For example, they are nonmagnetic and have low electrical and thermal conductivity. They can be molded directly to dimensions without the development of any considerable internal stress. Their high corrosion resistance should make them of value for applications in chemical plants and installations subject to acid fumes or to salt air. Other applications may benefit from the strength-to-weight ratios of plastics, which are often higher than those obtained with spring making metals. Also, the broad range of transparent and colored materials that can be used makes striking decorative effects possible.

The plastic springs are molded by drawing resin-soaked glass fibers through vinyl copolymer tubing and wrapping the loaded tubing in a helix around a mandrel. After curing in an air-circulating oven, the tubing is removed, leaving a solid plastic spring reinforced with glass fibers.

The most successful procedure uses vinyl chloride-vinyl acetate copolymer tubing having a $\frac{1}{4}$- or $\frac{3}{16}$- in. inside diameter and a $\frac{1}{8}$-in. wall thickness. Lengths of glass rovings are formed into a loose yarn, without twist, which is then doubled back to make a U-shaped bundle. This bundle is placed in a trough partly filled with liquid resin and is immersed until thoroughly soaked. A steel wire is threaded through the tubing, attached to the bend in the U-shaped bundle, and used to pull the bundle through the tubing. Considerable force is applied to the wire by means of a small windlass.

When the tubing is completely filled, the wire is disconnected and the ends of the tubing sealed with hose clamps. The tubing is wound on a mandrel and the entire assembly is placed in an oven for curing under the conditions specified by the resin manufacturer. After re-

[1] For further details, see "Reinforced Plastic Springs," by Frank W. Reinhart and Sanford B. Newman, *Product Engineering* (in press).

moval of the tubing, post curing is performed as required, and the springs are cut and ground to the finished length.

Of the resins studied, the most promising were epoxides and polyesters. These resins are better suited for spring applications on the basis of torsional moduli, temperature sensitivity, and ease of handling. In general, the polyesters are more sensitive to elevated temperatures than the epoxy resins, but the epoxy materials vary widely in this respect.

Springs made from polyester resins with glass fiber reinforcement showed good energy recovery properties after short periods of storage at −40° F., but these properties were adversely affected by 13 days' exposure to 135° F. while under stress. The best results were obtained with an epoxy resin cured with m-phenylenediamine. Springs made of this material had torsional moduli of rigidity of the order of 1.0×10^6 lb/in^2. In 2-in. lengths they deflected $\frac{1}{2}$ in. under a static load of 25 lb. Three-inch springs of this type retained 40 per cent of their original energy when compressed to their solid length and stored at 135° F. for 13 days.

When the glass-epoxy springs were tested for recoverable energy a second time, higher values were invariably obtained. This characteristic "work tempering" of the epoxide resins was also demonstrated in torsion tests on nonreinforced plastic rods. An improvement of as much as 120 per cent over the original available energy was indicated. Preloading the epoxide spring during storage or within 30 days before use would make it possible to utilize this property of the material.

The mechanical and thermal properties of the plastic springs can be varied widely by proper choice of materials and dimensions. For example, stiffer and probably more brittle springs result when the glass content is increased. At the same time a high degree of homogeneity in static properties can be achieved. Under a load of 25 lb., 100 of the epoxy-resin springs had an average deflection of 0.53 in. with a standard deviation of 0.05 in.

ACCELERATED AGING OF RECORD PAPERS

The National Bureau of Standards has recently completed a 26-year experiment comparing accelerated aging tests of record papers with the effects of normal aging over long periods. Results of physical and chemical tests on papers aged in both ways show a fair correlation between natural and accelerated aging.[1]

Scientists have long sought, with varying degrees of success, to predict the useful life of materials by means of accelerated aging tests in the laboratory. While many such tests have been designed for

[1] For further details, see "Accelerated Aging of Record Papers Compared with Normal Aging," by William K. Wilson, Jack L. Harvey, John Mandel and Thelma Worksman, *TAPPI*, Vol. 38, p. 543 (1955).

various kinds of materials, correlation with results obtained in actual service has not always been satisfactory. Obviously, a reliable means of estimating the useful life of paper should be of value to libraries, archival agencies, life insurance companies, and other institutions keeping long-term records.

Accelerated aging of papers is usually carried out by holding the papers at an elevated temperature for some time. Since the rates of many chemical reactions are approximately doubled with each 10°-C. rise in temperature, the use of higher temperatures provides, in general, a means of accelerating the aging process. However, this does not necessarily mean that the same reactions occur at higher temperatures as do at room temperatures. The correlation between accelerated and natural aging can only be determined by actual comparison of specimens aged in both ways. This requires storing specimens of the materials for several years and testing them from time to time.

To provide the data for such a comparison, the Bureau in 1928 measured the physical and chemical properties of a series of papers before and after accelerated aging at 100° C. for 72 hr.[2] At that time additional specimens of the papers were stored for normal aging and future testing. After four years[3] and again after eight years[4] of normal aging, the stored specimens were tested for folding endurance; tensile strength; water-soluble acidity; pH; alpha-, beta-, and gamma-cellulose; and copper number. The chemical tests were repeated after 22 years and the physical tests after 26 years.

The papers included in the study were mostly commercial writing papers ranging in fiber content from 100 per cent rag to 100 per cent sulphite. Four soda-sulfite book papers were also included. The specimens were not exposed to unusual conditions of temperature, humidity, or air pollution during storage.

Statistical analysis of data on folding endurance gave some indication of the relationship between natural aging and accelerated aging. No quantitative conclusions could be drawn, as the manufacture and testing of paper had not progressed far enough in the late twenties to permit the proper control of variables in the selection of samples. However, logarithmic plots of natural and accelerated aging versus time indicated reasonably good correlation.

The chemical test data showed fair correlation between normal aging and accelerated aging of all papers except the best rag papers. Changes in the latter were almost negligible after 22 years of normal

[2] "A Study of Purified Wood Fibers as a Paper-Making Material," by Royal H. Rasch, *J. Research NBS*, Vol. 3, p. 469 (1929) RP 107.

[3] "Comparison of Natural Aging of Paper with Accelerated Aging by Heating," by Royal H. Rasch and B. W. Scribner, *J. Research NBS*, Vol. 11, p. 727 (1933) RP 620.

[4] "Comparison of Accelerated Aging of Record Paper with Normal Aging for 8 Years," by B. W. Scribner, *J. Research NBS*, Vol. 23, p. 405 (1939) RP 1241.

aging, whereas small but definite changes had occurred after accelerated aging.

The tensile strength of the papers decreased very little with either accelerated or normal aging. However, such changes as were observed did appear to show a correlation between the two types of aging. As the folding endurance decreased sharply for all types of papers, it is apparent that much greater change occurs in the fatigue characteristics of the papers than in the strength.

Any consideration of the permanence of paper must take into account both the stability of the cellulose and the structural stability, or stability of sheet formation. Cellulose permanence is independent of the sheet structure, but paper permanence depends both on cellulose stability and on the permanence of the sheet formation. Chemical tests are a measure of cellulose permanence while physical tests show the structural stability of the paper.

The chemical test data on the rag papers indicate very little deterioration in 22 years, but the folding endurance decreased, on the average, to less than half of the original value. It thus appears that structural stability of the paper sheet decreases much more rapidly with aging than does the stability of the cellulose molecule.

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JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in this JOURNAL within the next few months:

COULSON, THOMAS: Some Prominent Members of The Franklin Institute. No. 3. Matthias William Baldwin
ATKINSON, CYRIL P. AND L. O. HEFLINGER: Subharmonic and Superharmonic Oscillations of a Bilinear Vibrating System
MOORE, E. F. AND CLAUDE E. SHANNON: Reliable Circuits Using Less Reliable Relays
ROBERSON, R. E. AND D. TATISTCHEFF: The Potential Energy of a Small Rigid Body in the Gravitational Field of an Oblate Spheroid
DAWSON, EDWARD: Pressure Transmission in Liquid Films
ROSENBERG, R. M.: A Pursuit Problem

Compact Klystron.—Probably the smallest klystron ever built, the new VA-97 has recently been introduced by Varian Associates, manufacturers of klystrons and related microwave equipment. The VA-97 is a rugged reflex klystron suitable for use as the local oscillator tube for airborne radar and similar applications. In the VA-97 reflex klystron, the integral external tuning cavity design has been carried up to a frequency range of 34.0 to 35.6 kMc. According to the manufacturer, this design combines outstanding environmental performance with excellent electrical performance. It is tough and reliable and is rated for use at any altitude without pressurization. At only 400 v beam voltage, it provides adequate power and electronic tuning for radar local oscillator service. The VA-97 features a slow-motion tuner free from mechanical backlash.

The manufacturer states that the VA-97 klystron was developed to allow maximum latitude in equipment design. Its compact size and light weight occupies a minimum of high premium aircraft space.

A companion klystron, the VA-94, is also available. It provides comparable performance characteristics in a frequency range of 16.0 to 17.0 kMc.

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NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION
ELLICE McDONALD, M.D., FOUNDER WILLIAM G. BATT, D.Sc., DIRECTOR

DIFFERENCE IN RESPONSE OF ANIMAL AND PLANT CELLS TO MITOTIC POISONS

BY

IRVING GALINSKY

Animal and plant cells appear to differ in their response to mitotic poisons and this difference seems to lie mainly in the type of abnormal metaphases. Levan (**1**) designates these as the "animal" type of c-mitosis and the "plant" type of c-mitosis. The animal and plant types differ in one main point, namely, the centromeres of the animal type do not divide and the c-metaphase chromosomes go directly into the c-telophase stage. In plants treated with colchicine or other mitotic poisons the centromeres of the metaphase chromosomes divide to give the c-anaphase stage in which the daughter anaphase chromosomes are separate from each other before going into the c-telophase stage.

The animal type of c-mitosis was observed by Levan in colchicine-treated Landschütz I and Ehrlich ascites tumors. The same type of c-mitosis was observed by Galinsky to occur in the untreated Krebs 2 ascites tumor. In the Krebs 2 ascites, the abnormalities become prominent about eight days after the mice are inoculated with the tumor. The evidence obtained from observations of these mitoses, supports the view that this is due to the accumulation, in the ascitic fluid, of substances which act as mitotic poisons. The mitotic abnormality that occurs most frequently is the failure of the centromere of the metaphase chromosome to split, so that the transition from the c-metaphase to the c-telophase stage is a direct one.

However, in a small proportion of the Krebs 2 ascites cells the centromere divides to produce the "plant" type of c-mitosis, as also observed by Levan in his material. The reverse has been reported to occur occasionally in plant cells treated with mitotic poisons (**2, 3**), that is, the centromere does not divide until the resting nucleus has been formed.

The difference, therefore, seems to be a quantitative one rather than a qualitative one. Levan concludes, "It is possible that a lower degree of poisoning favors the division of the centromere. It should be observed that there is a pronounced difference in toxicity of the colchicine between plant cells and animal cells." Galinsky (**4**) came to the same

conclusion from results obtained with onion root tips treated with sodium and potassium phosphates. He observed that the higher concentrations delayed the division of the centromere and produced many "star-shaped," "animal" type c-metaphases which directly went into the telophase stage. Lower concentrations of the phosphate resulted mainly in the "plant" type, scattered c-metaphases in which the centromere divided to give the c-anaphase stage.

The evidence seems to indicate that the centromere of animal chromosomes is more easily prevented from dividing than is the centromere in plant chromosomes. This would then account for the mitotic differences obtained in the treatment of animal and plant cells.

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BOOK REVIEWS

RADIATION BIOLOGY: VISIBLE AND NEAR-VISIBLE LIGHT (VOL. III), edited by Alexander Hollaender with the cooperation of Sterling B. Hendricks. 765 pages, tables, figures, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $10.00.

The publication of this third and final volume represents the conclusion of a tremendous amount of cooperative work by many contributing scientists in bringing to fruition what is certain to be a landmark in the field of radiobiology. This volume discusses the effect of visible and near-visible light including relatively recent research findings on photosynthesis, photoperiodism and other topics important to biological growth and energy. It consists of 15 chapters written by 21 specialists in the field.

In the first chapter, "Energy Exchange in Photoreactions" by R. Lumry and H. Eyring, there is a detailed presentation of the general principles of energy-exchange processes which occur in chemical and physical systems. The next chapter by J. R. Platt entitled "Electronic Structure and Excitation of Polyenes and Porphyrins" but which, according to the author, might be entitled: "Why grass is green, butter yellow, and blood red" deals with the compounds responsible for these colors. To understand the colors—which are probably the most important classes of biological pigments—requires not only the knowledge of electronic structure of the compounds but also their connection with the absorption of light in the visible and near-ultraviolet regions. Chapter 3 by R. B. and A. P. Withrow and entitled "Generation, Control, and Measurement of Visible and Near-visible Radiant Energy" is an exposition of modern developments and the most important of the fundamental principles and technics of application of visible and near-visible radiant energy to biological problems. This extremely valuable chapter is the longest in the book (133 pages) and very likely will be extensively consulted by the researcher. The next five chapters deal largely with the fundamental process of photosynthesis or important aspects of it as follows: "Energy Efficiency in Photosynthesis" by F. Daniels; "The Mechanism of Photosynthesis" by E. C. Wassink; "The Absorption, Action, and Fluorescence Spectra of Photosynthetic Pigments in Living Cells and in Solutions" by C. S. French and V. M. K. Young; "Chlorophyll Formation and Accumulation in Plants" by J. H. C. Smith and V. M. K. Young; "Nitrate Reduction" by H. Burström. These chapters present a veritable storehouse of information dealing with the principles of photochemistry, the mechanism of the photosynthetic process, measurements of energy and chemical changes involved in the process, practical applications of photosynthesis, tracer experiments, precursors of chlorophyll, temperature and nutritional effects on chlorophyll formation and accumulation, mechanism of chlorophyll biosynthesis, and the indirect actions of light on the nitrate reduction. In addition to a detailed presentation of recent information on photosynthesis, there is a prediction of the future when Dr. Daniels states, "At the time of this writing (1952) it has not been possible to produce this reaction (photosynthesis) in the absence of chlorophyll or in any way outside the living plant, but there seems to be no theoretical reason why photosynthesis cannot at some time be accomplished *in vitro* as well as *in vivo*." Chapter 9 by F. W. Went is entitled "Phototropism"—the response demonstrated by plants when they bend within a light gradient. Conveniently following is "Photoperiodism" by H. A. Borthwick, S. B. Hendricks, and M. W. Parker. This chapter deals with the discovery of photoperiodism, photoperiodic responses of animals, action spectra, pigment and the photoreaction, flowering in relation to auxins and, among other facets of the problem, reviews on photoperiodism. Remaining chapter headings with authors will suggest the broad scope of this volume: "Seed Germination" (M. Evenari); "Viscosity, Permeability, and Protoplasmic Streaming" (M. G. Stalfelt); "Electrical Phenomena in Vision" (L. A. Riggs); "In-

vertebrate Photoreceptors" (L. J. and M. J. Milne) and the concluding chapter, "Photodynamic Action and its Pathological Effects" (N. T. Clare).

The volume is certain to be eagerly welcomed as a fundamental collection of comparatively recent material on topics important to biological growth and energy. It is unfortunate that most of the 15 manuscript chapters (10) were received by the editor for publication as early as 1951; the latest having been received in 1953. In a work of this kind, however, a certain amount of delay from manuscript to publication is understandable. Some of the authors have corrected this publication delay by addenda containing latest information at the conclusion of the chapters.

This reviewer has looked expectantly to the publication of each of the four books making up the three volumes all of which are reviewed in this journal. They are indispensable to those in the field of radiation biology. Since treatment of material overlaps among the three volumes, all should be consulted for full coverage of a subject. At $35.50 for the set, it is a great bargain.

RALPH WICHTERMAN

VISTAS IN ASTRONOMY, Volume I, edited by Arthur Beer. 776 pages, illustrations, $8 \times 10\frac{1}{2}$ in. London, Pergamon Press, Ltd., 1955. Price, $28.00.

The "age of specialization" which matured to full bloom in the mid-Twentieth Century has of necessity restricted the scientist to a full and complete knowledge of only a fraction of the various facets in his special discipline. As a result the scientist in general and the astronomer in particular have been hard put to keep abreast of all research and developments which have recently materialized.

Reading all the literature is not the solution. First, because there are so many journals available in so many languages it is just impossible to read it all and second, many significant and current developments have not found their way into the literature. As a result there should be available a source where the current achievements, potentialities and trends are disclosed. Such a source is *Vistas in Astronomy*.

This is Volume I of a two-volume series. The book is divided into seven sections. The first deals primarily with international cooperation in astronomy. Various facets of this theme are covered by thoroughly competent astronomers. Section 2 consists of 12 articles dealing with the history and philosophy of astronomy. These range from historical representations of 2100 B.C. to modern cosmological concepts. In this section is found a truly significant paper on "Fact and inference in theory and observation" by the highly articulate H. Bondi. Dr. Bondi owes it to astronomy to elaborate on this section and remove the dogmatism that one finds in an old established science. This paper should be read and reread by every graduate student.

"Dynamics" is the title of section 3. As the title indicates it deals with the motions of the celestial bodies. Celestial mechanics, a particular branch of dynamics, has been neglected too long. Now with the earth-satellite program in full swing there will come a resurgence of interest in this subject.

Section 4, devoted to theoretical astrophysics, contains every facet of the subject the name implies while Section 5 is devoted to modern astronomical instruments. These range from clocks to computing machines in astronomy and from the conventional telescope to the electron telescope by Lallemand.

Radio Astronomy, the newest facet in astronomy is considered in Section 6 and the final section is devoted to Solar Physics.

This book is dedicated to Prof. F. J. M. Stratton, Emeritus Professor of Astrophysics at the University of Cambridge, and one of the most colorful workers in the field. As a tribute to this figure of international stature almost every country where active astronomical activity exists has contributed to this authoritative work. These contributions, in many languages, have all been translated in English with the editor faithfully rendering them so as to maintain the integrity of the theme.

In a volume of 192 contributors it is manifestly impossible to review this in the manner of a book devoted to a single theme. The task, if attempted, would have been made more difficult because of the controversial nature of some of the contributions, Of this the editor says, "Everyone familiar with our subject (astronomy) knows that it is a field in which controversy cannot be

avoided; indeed, at the frontiers of expanding knowledge, controversy is most desirable. Contradictory views will therefore sometimes be found here side by side with one another."

While this reviewer dislikes the new innovation of a book made up of separate monographs, he feels this one has been catalysed by the editor into a coherent unity. Authority permeates the book from cover to cover. Time will indicate Volume I of *Vistas in Astronomy* as a monumental work and a credit to those who conceived and executed the project. It is one of the most provocatively stimulating books on the astronomical horizon. I. M. LEAVITT

INTRODUCTION TO PLASTICITY, by Aris Phillips. 230 pages, diagrams, 6×9 in. New York, The Ronald Press Company, 1956. Price, $7.00.

The book *Introduction to Plasticity* is a text for a beginning student and practicing engineer on the subject of plasticity as applied to structures. Any advanced student, graduate student and practicing engineer who has studied engineering mechanics, structural analysis and theory of elasticity is able to study this text without difficulty.

In these modern times so much work is being done and interest shown in rockets, guided missiles and high speed aircraft in which the structures do not require long life and excessive strength that a knowledge of the collapse load is of primary importance. To the stress analyst in these fields Professor Phillips' book should prove of great value.

The text includes a discussion of stress-strain diagrams in the elastic and plastic region, the calculation of collapse loads of trusses, beams, and frames, bending of beams with and without axial forces, curved beams, collapse load under combined stresses as experienced in thick wall spherical shells, tubes and rings under internal pressure, wire drawing and wire extrusion. The final chapter deals with strain hardening under combined stresses. In the Appendix are the proofs of theorems used in the text and answers to problems. In various chapters are data of use to the stress analysis.

Professor Phillips' book should be of real value to the young engineer and the older practicing engineer desiring to keep abreast of the trends in structural design and analysis. A. O. BERGHOLM

HISTORY OF AMERICAN TECHNOLOGY, by John W. Oliver. 676 pages, 6×9 in. New York, The Ronald Press Company, 1956. Price, $6.50.

One of the defects in the manner in which history is taught is that too much attention is devoted to the achievements of dynasties and statesmen, while only passing reference is made to the work of the more influential scientists and inventors. For it is the latter who create the concrete conditions upon which the rulers and statesmen must construct their abstract policies. Dr. John W. Oliver, emeritus professor of history at the University of Pittsburgh, has attempted to remove this reproach from American history by providing an adequate technical background. In so far as the task could be accomplished in a single volume, although generous in bulk, he has succeeded in his purpose.

The plan of the book is ambitious since it covers the whole area of science and technology. Its chronology is divided into four parts, the first of which treats with the years before the Revolution; the second, from the Revolution to the Civil War; the third from the Civil War to the year 1900; and the final section brings the story down to the present time. These are appropriate divisions since they permit a review of the four principal periods of developments that have distinctive features: the transplanting of tools, skills, and technologies; the laying of the foundation of American supremacy in technology; the awareness of science and the triumph of machine technology; and the science and technology of the leading world power.

The book is a mine of information covering this wide area that includes agriculture, communication, transportation, building, etc., as well as manufacturing technology. Having such an extensive field to cover it is not to be expected that individual achievements can be described fully, but their significance is not overlooked, and they are presented in the right proportions. All the familiar names appear and a great many that are unfamiliar are given the prominence they deserve. Indeed, after careful search-

ing, the only considerable omission this reviewer could find was that of Thomas Godfrey, who deserved mention as the inventor of the sextant, an honor he shared with Hadley. A few words might have been devoted to standardization, with a bouquet thrown in the direction of The Franklin Institute for the lead it gave in the matter of screw threads. But these are small flaws in a work that has all the traces of painstaking, sustained research balanced by fine judgment.

In a book of this nature an adequate index is essential. Repeated tests have shown that the index to this volume responds creditably to the reader's demands. Altogether, we feel, this will be a welcome addition to those books that bring learning and energy to the study of the place of technics in human culture.

T. C.

DYNAMICS OF MACHINERY, by James B. Hartman. 283 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $7.50.

Dynamics of Machinery is published as one volume in the McGraw-Hill series in Mechanical Engineering. This book is intended for the advanced undergraduate and graduate student, and the practicing engineer in industry who desires to study formally some advanced methods of analysis. The users of this book should have a knowledge of engineering college mechanics, dynamics, vibrations and mathematics including the calculus. Besides this a knowledge of differential equations and Laplace transforms would be very helpful.

The opening chapter deals with some principles of statics including vector notation. The second chapter similarly treats with dynamics again using vector notation. Chapter three deals with inertia forces of various mechanisms including the four bar linkage. The following chapters deal with vibrations, balancing of machinery, engine dynamics, mechanical transients, dynamics of automatic-control systems.

This book can be commended for its use of more advanced mathematical treatment and notation. The chapters on mechanical transients and dynamics of automatic-control systems should be of timely interest to the practicing engineer and the advanced student. The only question that may arise is the brevity of the book considering the subjects covered. The latter point may be a source of concern when the student attempts to follow the rather brief mathematical developments that are prevalent.

A. O. BERGHOLM

HIGH-TEMPERATURE TECHNOLOGY, edited by I. E. Campbell. 526 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Limited; 1956. Price, $15.00.

During recent years, there have been numerous symposia on high temperature technology and a large number of papers in the published literature. However, there have been very few detailed summaries of the subject. The present text fills the need for such a summary. It deals primarily with materials for use above 1500° C.

The book is divided into four sections. Part I is a relatively short introduction outlining the need and the development of modern refractory materials. Part II covers the materials used for high temperatures. Parts III and IV discuss the methods and measurements involved in high temperature technology.

Since the temperatures under discussion are above 1500° C., there is not much information concerning metals. Only 12 papers are devoted to this phase of high temperature materials so that the metallurgist is apt to be disappointed. However, the sections on oxides, borides, carbides, silicides, nitrides, sulfides, and cermets are quite comprehensive. These sections tabulate many of the physical, chemical, mechanical, electrical, and thermodynamic properties of the refractory materials. The data are up-to-date and comprehensive and should form a valuable reference source for those interested in this field. The section on methods includes a brief and superficial treatment of the sintering process. The remainder of this section is devoted to the various means of achieving high temperatures. A fair amount of detail is given in the chapter on arc furnaces. Several good schematic diagrams of furnace designs are shown.

The last section of the book is devoted to measurements. It covers temperature measurement, mechanical and physical properties,

high temperature microscopy and high temperature X-ray diffraction. In the chapter on temperature measurement, a tabulation of the properties of some of the refractory oxides is given. Some of the data do not agree with those presented in an earlier chapter on the oxides. In fact, none of the melting points agree; for example, one table lists thoria as melting at 3030° C. and the other at 3300° C.

One of the outstanding chapters in the book is that devoted to the measurement of physical properties. Thorough descriptions and excellent schematic diagrams are given describing the measurement of such properties as melting point, phase equilibria, vapor pressure, thermal conductivity, thermal expansion, etc.

The book contains a subject index and an author index. Extensive bibliographies are given at the end of each chapter. In general, it is well edited and should serve as an excellent reference book for those involved with high temperature refractory materials.

R. L. SMITH

THE CASTING OF STEEL, edited by W. C. Newell. 599 pages, illustrations, 6 × 9¾ in. London, Pergamon Press, 1955. Price, $15.50.

The Casting of Steel is edited by W. C. Newell, formerly head of Steel Castings Division, British Iron and Steel Research Association. Published in England, it has the British style and format to which many American readers are not accustomed.

The topics covered in the text are: Properties of the Liquid Metal, The Mechanism Solidification, The Melting Process, The Use of Refractories, Pattern Making, Mould Preparation and Core Making, Casting into Sand Moulds, Centrifugal Formation of Castings, Investment Casting Process, The Heat Treatment and Properties of Steel Castings, Mechanical Testing, Radiography, and Non-Destructive Testing.

By reviewing the contents it is observed that this text is a handbook that is of value to those in the steel foundry business and the young engineer entering the field. To the American, this book should be of interest but he will find the practices and equipment described will be strange. Regardless of nationality, the chapters on Non-Destructive Testing and Radiography should be of interest, and particularly the last item which is developing so rapidly at the present time.

A. O. BERGHOLM

FREQUENCY RESPONSE, edited by Rufus Oldenburger. *372* pages, diagrams, 8½ × 11 in. New York, The Macmillan Co., 1956. Price, $7.50.

This book consists of a worthwhile collection of articles dealing primarily with the analysis and synthesis of feedback control systems using frequency response methods. Most of the papers were presented at the 1953 ASME Frequency Response Symposium, with the result that twenty of the twenty-eight articles included were published in the *ASME Trans.*, Vol. 76, no. 8 (1954). The discussion of the various papers and the authors' remarks are also included.

The material in the book ranges from the basic ideas of frequency response to the applications of such methods to nonlinear systems. Some material on experimental techniques is also included. Those interested in the historical growth of the field will find quite helpful the introductory articles and the lengthy bibliography of papers and books published up to 1953. Those with no previous experience with feedback control systems will find the book almost as readable as an introductory text in the field, although less complete. Because of its essentially tutorial nature, however, *Frequency Response* will probably not provide the electrical engineer who has been working with control systems for the past few years with new ideas or techniques.

PRESTON R. CLEMENT

ELECTRONIC DATA PROCESSING FOR BUSINESS AND INDUSTRY, by Richard G. Canning. *332* pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1956. Price, $7.00.

In what was a field starving for literature a short time ago, a large amount of material is now becoming available. This book is aimed at management with the hope of explaining what data processing is, what its significance is to a company and how management should go about investigating it. The author's concept of data processing relates to management decision-making opera-

tions for control purposes including a feedback on progress.

Mr. Canning discusses the characteristics of the major components of data processing equipment and the programming of a department store application. This is followed by a detailed discussion of how the systems engineer would tackle this problem and what is contributed by an operations research study. The importance of such studies cannot be over-estimated. Some elements of management may be surprised at the extent of these studies that are required or at the fact that some long established company procedures will have to be revised to accommodate data processing. It can be almost catastrophic to install blindly large and costly equipment of this type in an attempt to keep up with the competition. A careful study of this book should avoid this. Finally the author treats the systems engineers' work up to the time a proposal for action is presented to management.

Types of equipment are treated for applications such as file maintenance. The emphasis throughout the book is on how to study the needs of a company for electronic data processing equipment and wisely avoids appraisal of existing equipment which, as Mr. Canning says, is soon out of date in this fast moving field.

DONALD B. HOUGHTON

MACHINE DESIGN, by Joseph Edward Shigley. 523 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $7.75.

Few books have been written that stress creative thinking in the learning process of the engineering student as this book on machine design does. While it deals with the mechanical field, it is conceivable that a similar pattern may be followed in many other types of technical, academic and commercial fields.

The objectives are clearly defined in the preface and are religiously followed in the text. A background of mathematics and elementary machine design should precede the use of this text, but the first part of the book deals with fundamentals.

Part Two deals with the application of those principles with an excellent presentation of steps to be followed which may be equally applicable to other fields of creative work. Many of the problems and illustrations in the book require decisions to form a basis for solution and, while some problems have but one correct answer, others depend upon the assumptions made for a successful product design. Evaluation of results is presented by various means and the student who completes the course including this book should be in a capable and mature position that would otherwise require years of experience to acquire. EARL R. BOWMAN

SURVEYS IN MECHANICS, edited by G. K. Batchelor and R. M. Davies. The G. I. Taylor 70th Anniversary Volume. 475 pages, diagrams, 5½ × 8½ in. New York, Cambridge University Press, 1956. Price, $9.50.

Surveys in Mechanics was edited in commemoration of the seventieth birthday of Sir Geoffrey I. Taylor, 7 March 1956. The editors of this commemorative book selected ten subjects to be incorporated in the volume that G. I. was interested in and worked on during his life time. An authority in each subject was asked to contribute one chapter. Usually the articles give the historical background, the development and the present status of knowledge in the field. In addition to the ten chapters there is a short biography of G. I. Taylor that is of interest.

The subjects of the ten chapters are: "The Mechanics of Quasi-Static Plastic Deformation in Metal," "Dislocation in Crystalline Solids," "Stress Waves in Solids," "Rotating Fluids," "The Mechanics of Drops and Bubbles," "Wave Generation by Wind," "Viscosity Effects in Sound Waves of Finite Amplitude," "Turbulent Diffusion," "Atmospheric Turbulence," and "The Mechanics of Sailing Ships and Yachts." A bibliography is attached to each chapter.

The reader will find each chapter to be extremely interesting and stimulating regardless of his scientific background and proficiency. There are some mathematics involved, but the amount is small and should not deter one from reading this book. The editors should be commended in compiling such an interesting and readable book.

A. O. BERGHOLM

PUBLICATIONS RECEIVED

The following books have been received, but space limitations prevent publication of reviews of them.

ANALYTIC GEOMETRY, by Paul K. Rees. 237 pages, diagrams, 6 × 9 in. Englewood Cliffs (N. J.), Prentice-Hall, Inc., 1956. No price.

GENERAL CHEMISTRY, by John Arrend Timm. Third edition, 636 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $6.00.

INDUSTRIAL ELECTRICITY. VOLUME I. DIRECT CURRENTS, by Chester L. Dawes. Third edition, 431 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $5.50.

INTRODUCTORY ORGANIC CHEMISTRY, WITH CERTAIN CHAPTERS OF BIOCHEMISTRY, by E. Wertheim and Harold Jeskey. Third edition, 476 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $5.50.

HOW TO CALCULATE QUICKLY, by Henry Sticker. 256 pages, 5 × 7¼ in. New York. Dover Publications, Inc., 1956. Price, $1.00 (paper).

THE EVOLUTION OF IGNEOUS ROCKS, by N. L. Bowen, with a new introduction by J. F. Schairer. Unabridged and unaltered republication of the first edition. 332 pages, diagrams, 5⅜ × 8 in. New York, Dover Publications, Inc., 1956. Price, $1.85 (paper); $3.50 (cloth).

COLOUR PRINTS, by Jack H. Coote. 325 pages, diagrams, plates, 5 × 7¼ in. London, The Focal Press, 1956. Price 25/, $4.95.

PHOTOGRAPHIC OPTICS, by Arthur Cox. Eleventh revised edition, 374 pages, diagrams, 5 × 7¼ in. London, The Focal Press, 1956. Price: 17/6; $5.75.

ORGANIC CHEMISTRY, by Louis F. Fieser and Mary Fieser. Third edition, 1112 pages, 6 × 9½ in. New York, Reinhold Publishing Corp., 1956. Price, $10.00.

PRECISION ELECTRICAL MEASUREMENTS. v.p., diagrams, 8½ × 11 in. New York, Philosophical Library, 1956. Price, $12.00.

A TREATISE ON SURVEYING, by Middleton and Chadwick. Revised sixth edition. Two volumes, 381 and 438 pages, diagrams, 5½ × 8½ in. New York, Philosophical Library, 1956. Price, $20.00 the set.

UNITED STATES ARMY IN WORLD WAR II. CHINA–BURMA–INDIA THEATER. STILLWELL'S COMMAND PROBLEMS, by Charles F. Romanus and Riley Sunderland. 518 pages, illustrations, 7 × 9¾ in. Washington, Department of the Army, 1956. Price, $6.25.

UNITED STATES ARMY IN WORLD WAR II. THE TECHNICAL SERVICES. THE MEDICAL DEPARTMENT: HOSPITALIZATION AND EVACUATION, ZONE OF INTERIOR, by Clarence McKittrick Smith. 503 pages, 7 × 9¾ in. Washington, Department of the Army, 1956. Price, $4.00.

CURRENT TOPICS

Computer to Test Jet Engine Performance.—A giant computing machine that could save up to two years in the costly development of advanced-design jet engines has been built for the Air Research Development Command by the General Electric Company.

One of the largest ever built, this unique analog computer will enable engineers to "fly" jet engines before they are built and to forecast performance characteristics of control systems for jet engines that are still on the drawing boards.

The computer will become the "brain" for a jet engine simulator that will reduce costly wind-tunnel testing and cut down the time and cost of flight-testing, both vital phases in the race to develop stronger and faster aircraft.

The jet engine simulator was developed and is being built by General Electric's Aircraft Gas Turbine Development Department under an ARDC contract. The computer for the simulator was designed and built by the company's General Engineering Laboratory.

"Flying" the computer, an engineer can become a "test pilot" and in a matter of a few hours can see how his blueprint engine reacts to such conditions as giving it the throttle at 20,000 feet altitude, 40° below zero air temperature and at a speed of 1260 miles an hour.

Previously it has not been possible to check out a pre-production engine control system on a sufficiently large number of engines of one type. In many instances, the engine is either not available and its expected performance data are lacking, or, if an experimental prototype of the engine is available, engineers have been unwilling to risk the expensive prototype in gruelling tests.

Checking out an experimental engine control system normally takes up to six months and costs several million dollars in the wind-tunnel and flight-test phases.

The machine works this way:

Engineers study the specific problem at hand and reduce it to mathematical form. They then convert it into equations. The computer works out these equations electrically and comes up with the answer in the time it takes an air molecule to pass through an actual engine.

The General Electric computer, which took six years to build, measures 35 by 100 ft. and is 12 ft. high. It covers a floor area of 2,635 sq. ft., roughly the equivalent of three average-sized homes, and uses 40 tons of air conditioning, equivalent to air conditioning 15 average-sized homes or 80 room air conditioners.

The vast amount of air conditioning is necessary to insure the accuracy of instruments and the efficiency of vacuum tubes, which are sensitive to abrupt temperature changes.

Its power source operates on 2300 volts of alternating current. Its 150-horsepower motor and four generators can develop 103 kilowatts, enough to run 30 to 50 average electric kitchen ranges with all burners and ovens lit.

The computer units contain 6000 vacuum tubes, 140,000 ft. of wire and 1700 indicating lights. It has 2750 knobs or dials, 8200 plug jacks, 300,-

000 soldered joints and 100,000 solderless connectors.

An estimated 85,310 man-hours went into the finished product, roughly equivalent to 42 man-years of work. Approximately 8350 hours (2½ man-years) was spent in the drafting stage, approximately 44,960 man-hours in engineering (23 man-years) and about 32,000 man-hours (16.3 man-years) in manufacturing and assembly.

Organic Moderated Reactor.—A new type nuclear power reactor designed and to be built by Atomics International, a division of North American Aviation, Inc., has been selected by American and Foreign Power Co. Inc., for installation in a Latin American country. The new reactor, called the OMR (Organic Moderated Reactor) will produce 10,400 kilowatts of electricity. The Latin American country where the reactor will be installed has not yet been determined.

Atomics International is currently building an experimental OMR type reactor for the Atomic Energy Commission as part of the Commission's nuclear power development program. This reactor will be in operation early next year at the Commission's national reactor testing station at Arco, Idaho. Final arrangements for installation of an OMR in Latin America will depend upon results of tests with this type reactor at Arco and certain other development work.

The OMR will use an organic fluid, a carbon-hydrogen compound, to transfer heat from the atomic fission process to a steam generator. The organic fluid will also serve to "Moderate" or slow down neutrons in the fission process to make more efficient use of the nuclear reactor fuel.

American and Foreign Power selected the OMR as one of two reactors that firm will install in Latin America. The other reactor, a different type, will be supplied by General Electric.

According to Henry B. Sargent, president of American and Foreign Power, in addition to factors of capital and operating costs, the two reactors were selected on the basis of safety, technical soundness of design, and the fact that the reactors can serve as prototypes for larger atomic power plants. Additionally, Sargent pointed out that both reactors could be used for nuclear research in the fields of medicine, food preservation, the production of radioisotopes and other work in the atomic energy field.

Compact, Industrial Oscilloscope.—Development of an extremely compact, low frequency industrial oscilloscope has been announced by Hycon Electronics, Inc., Pasadena, Calif. Harry A. Kirkpatrick, general manager, said the instrument is sensitive to both direct and alternating current voltages, and has identical horizontal and vertical amplifiers to assure accurate phase angle measurement. The new oscilloscope features usable response up to 500 kilocycles and is "flat" within three decibels at from 0 to 300 kilocycles.

It translates electrical phenomena into visual displays, making it possible to examine, qualitatively and quantitatively, the characteristics of the electrical circuit under test. Built-in calibration voltage enables the user to measure the amplitude of voltage while it is being observed.

The instrument is suitable for mounting in the standard 19-inch relay rack or cabinets, Kirkpatrick said. Other features include a flat face cathode ray tube to minimize distortion; high grade transformers and similar features to assure rugged durability; and a minimum of component parts, facilitating service and maintenance.

Hycon Electronics, a subsidiary of Hycon Mfg. Company, put the instrument through intensive laboratory and field tests prior to its introduction.

The oscilloscope was designed, Kirkpatrick said, to meet the varied requirements in the electronic, aircraft, guided missile, research, chemical, and educational fields.

Beamplexer Redesigned.—The Beamplexer, a fast electronic switch used to sample up to ten separate signal sources and put them out in sequence on a single line, has been thoroughly re-designed to meet testing requirements found in the field, it has been announced by the Electronic Instruments Division of Burroughs Corporation.

One of the most important applications of the new Beamplexer is as an accessory to the conventional single channel oscilloscope. By enabling this instrument to display up to 10 separate channels of information simultaneously, it provides the user with the advantages of multi-channel oscillography at a mere fraction of the cost of a multichannel scope. Individual position and amplitude controls for each of the ten input channels allow vertical positioning of all the signals on the face of the oscilloscope so that they may be superimposed or placed in any desired relationship. This direct visual comparison simplifies studies and measurements of wave shapes, phase relationships, amplitude and frequency, and generally increases the application and effectiveness of the oscilloscope.

Each of the Beamplexer's input channels is a.c. coupled to an individual triode amplifier which has gain control and a pedestal level control in its grid circuit. The cathode of each of the triode amplifiers is connected to a corresponding target of a Burroughs beam switching tube. As the beam steps through its ten positions, at adjustable speeds ranging from pushbutton to 100 kc., current is supplied to the tube connected to the particular target on which the beam is formed. In this way only one tube at a time is allowed to conduct through the output circuit.

Available as a standard rack-mounting unit or in its own cabinet, the Beamplexer is a portable instrument containing a regulated power supply for a standard 60 cycle, 115 VAC source.

The original Beamplexer was announced by the Burroughs division about a year ago as one of the applications of the then newly developed Burroughs beam switching tube. Several instruments which were manufactured to investigate field application proved a widespread need for the instrument in telemetering, internal combustion studies, vibration analysis, servo-mechanism phase measurement, neurological studies, and multiplexing. The new Beamplexer is the result of a re-design project to meet the practical needs of these fields, as well as to make the instrument more adaptable to many others.

Transistorized Light Meter.—Transistors have entered the camera shop for the first time with Servo Corporation's unique Servotherm Photometer, a transistorized light meter that will give direct, dependable and accurate readings over an unusually wide range—ultraviolet through infrared.

Germanium transistors developed for the new Photometer by Radio Development and Research Corporation, a subsidiary of Bogue Electric Manufacturing Co., make possible the compact size and portability of the precision instrument. The transistors are not only miniature in size, but they also reduce the power needs of the

entire Photometer. A few miniature batteries will produce enough electricity to operate the meter, and no external source of power is needed.

In addition, RDR's germanium transistors are virtually indestructible and will work under almost any conditions, making the operation of the Photometer independent of altitude and temperature changes.

The operation of the meter itself is simple. Film response and correction filters are internally duplicated and direct stop and shutter readings can be taken without scale changes, tables or computations. The versatile instrument is also applicable to problems in color separation and analysis.

The Servotherm Photometer is available through the Servo Corporation of America, New Hyde Park, N. Y.

Air Pollution Paths Traced.—Air paths associated with heavy pollution concentrations in the Los Angeles area have been traced by means of wind reports from a network of 64 stations in the Los Angeles Basin.

According to Edwin K. Kauper, senior meteorologist with the Los Angeles County Air Pollution Control District, trajectories have been constructed for heavy smog situations from 1952 to 1955, showing the routes taken by air parcels from the time they crossed the coast-line until they arrived at an air sampling station, loaded with pollution.

Flow of polluted air from one geographical area to another is an aspect of the air pollution problem that is of the utmost importance to any organization engaged in control activities.

Speaking before the 49th annual meeting of the Air Pollution Control Association, Kauper said that the Los Angeles County APC District early recognized the value of knowing how the pollution of the Basin behaved under the influence of the wind. Thus the network of wind-measuring stations was expanded in 1952 for the purpose of providing detailed wind data applicable to future zoning decisions.

Studies of the air movement on smoggy days indicate that the air aloft is not such that pollution could be brought to central L. A. from an area other than that determined by trajectory construction based on wind reports.

Evidence presented by these trajectories has forced a re-evaluation of pollution sources since nearly all cases indicated a flow over predominately residential rather than industrial sections of the Los Angeles Basin.

Electro-Hydraulic "Yes Man."—A two-armed electro-hydraulic "slave" that could perform sensitive mechanical tasks in New York at the bidding of its "master" in San Francisco has been developed by General Electric engineers. Nicknamed "Yes Man" by its creators, the new machine is the most dexterous master-slave manipulator yet devised. It is designed primarily to do the work of a mechanic in radioactive areas in which it is too dangerous for a mechanic to work.

"Yes Man" is actually twins—two almost identical pairs of mechanical arms and hands that somewhat resemble a dentist's drill rigs. The "slave" is a virtual twin to the "master," except that it has claw-like fingers on each hand and has no operator's seat. Its arms, elbows, wrists, hands and fingers are electronically geared to duplicate simultaneously the exact motions of the "master."

To perform a given mechanical task, the operator seats himself in the "master" seat, straps his own arms and fingers into the mechanical arms and

fingers, and watches the "slave" duplicate each motion he makes.

This latest advance in mechanical hands was developed and built in General Electric's General Engineering Laboratory for the company's Aircraft Nuclear Propulsion Department under contract with the U. S. Air Force.

The same laboratory earlier developed "O-Man," the world's largest and strongest mechanical arm.

"Yes Man's" forte is dexterity. It has two fingers and a thumb on each hand, plus wrists and elbows, to give it wide flexibility.

Instead of pushing buttons and controls, the operator actually goes through the desired arm and finger motions for the task at hand. Operating the "master" machine, he actually can "feel" the force encountered by the slave as it tackles a job.

This sensitive touch enables him to control the slave with high proficiency in handling a wrench or a screwdriver, pouring water, placing objects or virtually any other mechanical chore.

It permits him to perform intricate operations in hazardous radioactive areas remotely and at a safe distance.

Were the slave on the Atlantic coast and the master on the Pacific coast, "Yes Man" would require many telephone channels and, to see what he's doing, one TV channel. Despite the mileage, the operator in San Francisco could still "feel" what the slave in New York was encountering. Normally the distances are nowhere near as large and the problem of interconnecting master and slave is simpler.

This unprecedented design gives "Yes Man" a big jump and makes it more dexterous than any known manipulator. Although not the strongest manipulator ever built, it is the strongest "spatial correspondence" system yet devised—that is, the strongest manipulator giving the operator the advantages of controlling many complex motions in a "normal" manner without having to think about which push buttons must be operated to achieve the desired motions. It can lift up to 100 lb. in certain positions and 25 lb. in any position.

The system consists of two master arms, one left and one right, each with two fingers and a thumb; left and right arms on the slave with corresponding finger arrangement; two separate hydraulic pumps, a rack of electronic amplifiers, and two groups of electro-hydraulic valves.

The electro-hydraulic valves convert the electrical signal into hydraulic power, which in turn operates the slave and gives the master its ability to "feel."

The slave can be mounted on a variety of devices to extend its work area—overhead cranes, rollers, a dolly, remote control truck or whatever is practical to obtain access to its work.

The developmental model built in the Schenectady Laboratory is currently undergoing operational tests. G-E engineers are already experimenting to make the machine's hands even more dexterous.

Positive Temperature Coefficient Thermistors.—Successful fabrication of thermistor materials having large positive temperature coefficients was revealed recently by two Bell Telephone Laboratories engineers speaking at the IRE–RETMA–AIEE–WCEMA Electronic Components Symposium in Washington, D. C.

Harold A. Sauer and Steward S. Flaschen discussed the preparation of one material having a positive temperature coefficient from about − 50° C. to 110° C., with the coefficient reaching a value as high as 9%/° C. Another material of slightly different composition was found to have a positive tem-

perature coefficient from about 50° C. to 225° C., with a maximum coefficient of 14%/° C.

These materials in general consist of barium titanate or barium-strontium titanates to which small amounts of lanthanum have been added. By properly proportioning the various components, a wide variety of characteristics can be obtained. The resulting materials, after proper processing, are electrically and thermally stable over operating temperature ranges.

At voltages below which power dissipation causes no perceptible heating, these thermistors are ohmic in character, that is, the resistance does not change as the applied voltage is changed. Suitable ohmic contacts consist of an indium-mercury amalgam which is rubbed on at the desired contact point.

Instruments Equipped with "Sixth Sense."—Special instruments equipped with a "sixth sense" have been designed by General Electric Company engineers to help control complicated chemical manufacturing processes at the Atomic Energy Commission's Hanford plant.

R. J. Brouns, General Electric chemist employed at Hanford, described two such "sixth sense" instruments in a paper presented before the 1956 General Electric Analytical Conference. General Electric operates the Hanford plant for the AEC.

Both instruments measure properties of chemical solutions continually during production runs, helping operating engineers keep close control over results and helping obtain a uniform operation. Data from the instruments are available for immediate use and process correction. Formerly several hours were often necessary to obtain analytical results from laboratory tests.

One of the instruments described by Brouns uses a by-product of plutonium production as a help in gathering data necessary to control that production. Atomic rays given off by a radioactive isotope are used to measure properties of materials under processing. By measuring the amount of radioactivity absorbed by the material, technicians can determine changes in the flow of materials under process.

Another instrument adapted by General Electric engineers for their own purposes uses a light bulb for its yardstick and a photoelectric cell as a measuring device. The light is filtered to select the wave lengths best suited to the measurements wanted. The amount of light passing through the filter is known to technicians and the amount that reaches the cell after passing through the material to be measured is reported on an electronic recorder. The light absorbed by the material under process is the "key" to the property to be determined.

This device has been designed with an eye for use in measuring radioactive materials, so its advantages are found in being cheap, virtually unbreakable, acid and fume resistant and in the fact that it can be quickly installed. The control unit for the sensing device can be located some 50 to 100 ft. away.

Units such as these are the first step toward automation in chemical processing, Brouns told his audience. Today engineers must read the data collected for them by these "sixth sense" machines, and then make manual changes to correct for errors reported. Further development should lead to automatic control of these plants. Before this can be developed, however, chemical engineers have an expensive and time consuming job before them in collecting the needed data for deriving the equations that will make true automation possible.

Fume- and Odor-Removing Equipment.—An actual working model of a newly designed unit to control fumes, odors and microscopic solids was demonstrated by The Johnson-March Corporation, Philadelphia, at the annual convention of the Air Pollution Control Association, in Buffalo, N. Y.

The equipment, known as the Type A Hydro Precipitator Scrubber, is entirely different from all other types of dust-collecting scrubbers. It is engineered to remove low and sub-micron size particles from exhaust gases at extremely high efficiency.

In test situations, the new unit has performed at 99.8 per cent efficiency with dust loadings at five grains per cubic feet and 70 per cent of the material below five-micron particle size.

The model had glass doors and panels, and an illuminated interior to show construction features. Water was re-circulated through the unit to show the method of operation. Visitors saw how the violent water turbulence and vigorous scrubbing action wets, traps, separates and forces solid matter to the bottom . . . thus following the complete dust removal cycle.

The unit on display was a 500-cfm all-metal scrubber measuring 8 ft. high by 4 ft. long by 3 ft. wide.

Camera Records Instrument Readings.—An "eye" that never sleeps keeps a constant watch over vital instrument dials inside an atomic reactor control room at the vast Hanford plutonium plant.

The eye is the lens of a 16-millimeter motion picture camera mounted on a platform near the ceiling of the control room. Focused on the instrument panel, the camera grinds intermittently, filming a permanent record of instrument readings.

General Electric scientists and engineers have applied this creation of the film industry on an experimental basis to an important phase of operation of the plant which the company operates for the Atomic Energy Commission.

They believe that in the time of a breakdown or minor mechanical failure, reactor operators will be too occupied with their immediate problems to recall the exact position of various dials just before or immediately after the incident.

To make certain this important information is recorded, the camera retains the readings on "slowed-down" movie film, the result of exposing film frames at established intervals. Exposure of a frame every few seconds or minutes, rather than many frames a minute as in ordinary moving pictures, speeds the apparent action of instrument dials when the finished motion picture is viewed. The same technique has been applied to speed the opening of a flower and the movement of tides.

The Hanford apparatus includes a 16-millimeter motion picture camera and an interval timer set to trigger the camera and expose film at the rate of one frame every second to 20 minutes. During the crucial startup of a nuclear reactor, the timer is set for 10-second intervals. As the chain reaction within the reactor approaches normal operating level, the camera timer is cut back to expose fewer frames per minute until it is geared to expose a frame of film every 15 minutes. An automatic timer turns on lights just before the shutter is opened to provide sufficient illumination. The slightest deviation in the chain reaction process can be detected.

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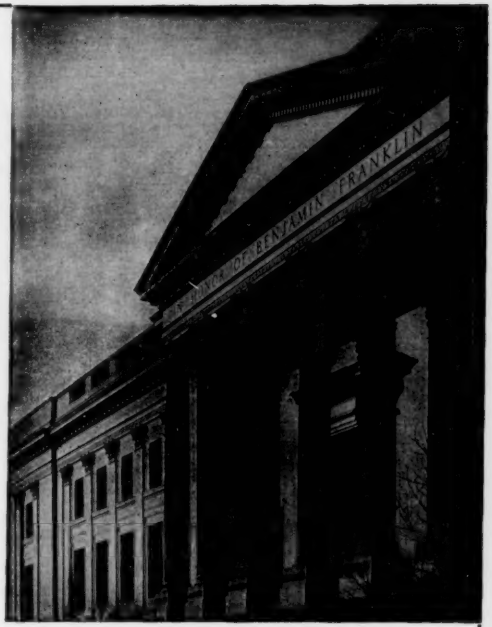

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